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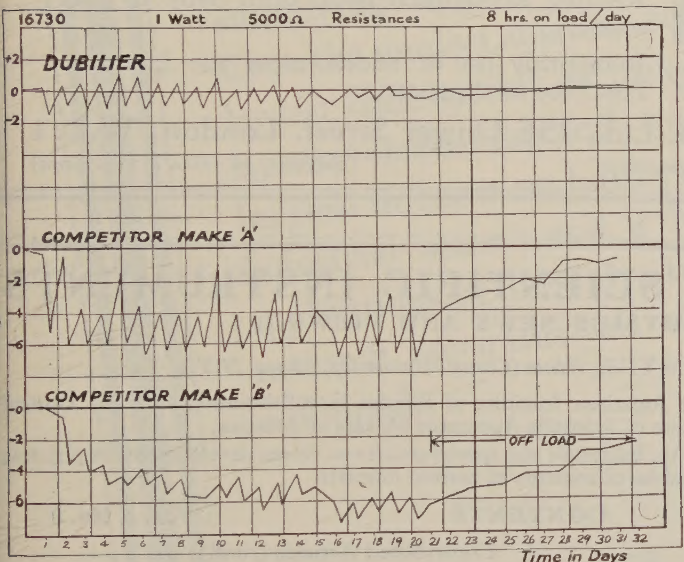

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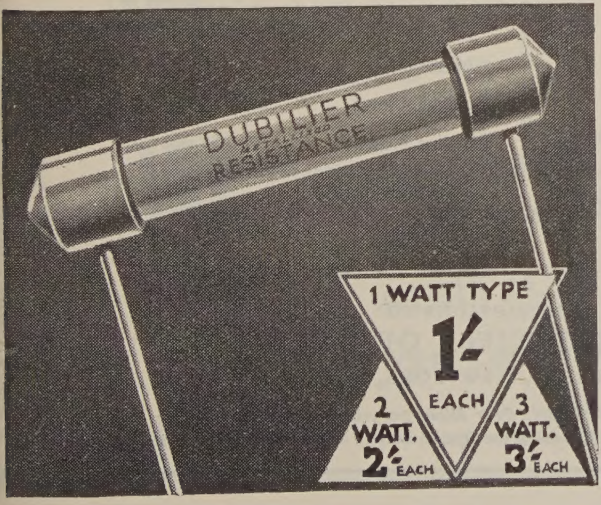

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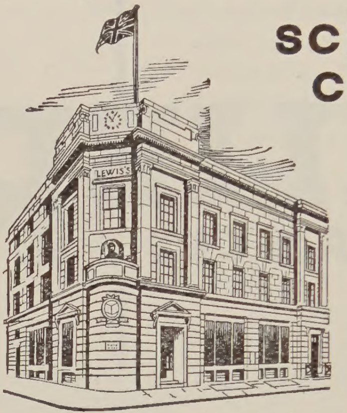

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"A method of measuring the specific heats of poor conductors," by W. G. Marley, M.Sc., page 591.

For 536.63:536.62 *read* 536.63   536.62.

THE PROCEEDINGS OF
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535.338:546.48

HYPERFINE STRUCTURE IN THE SPARK SPECTRUM OF CADMIUM

E. GWYNNE JONES, Beit Scientific Research Fellow, Imperial College of Science and Technology, London

Communicated by Prof. A. Fowler, F.R.S., June 15, 1933. *Read July* 7, 1933

ABSTRACT. The Cd II spectrum has been investigated in the region λλ 4200–8500 with Fabry-Perot etalons. Of the thirteen lines examined, only λ 8067 has any structure. From the structure of the $6\,^2S_{\frac{1}{2}}$ term it is calculated that the $g\ (I)$ factor of the odd isotopes is $-1{\cdot}25$ proton magnetons.

§ 1. INTRODUCTION

In the spectra of cadmium, only four arc lines have been thoroughly investigated for hyperfine structure. Michelson proposed λ 6438 ($5\,^1P_1$–$6\,^1D_2$) as the standard of wave-length measurements on account of its lack of structure, and the visible triplet λλ 4678, 4800 and 5086 ($5\,^3P_{0,1,2}$–$6\,^3S_1$) has long been known to possess satellites. The structures of these lines were accurately measured and explained by Schüler and Keyston[*], who assigned the nuclear moments $I = 0$ to the even isotopes (mass numbers 110, 112, 114 and 116) and $I = \frac{1}{2}$ to the odd isotopes (111 and 113). The present investigation was carried out in order to examine the lines of Cd II that fall in the region λλ 4200–8500, where a silvered interferometer can be used to best advantage.

§ 2. EXPERIMENTAL RESULTS

The spectrum, which had already been analysed by von Salis[†] was excited in a Schülerlamp of the hot cathode type[‡], in which the required lines were very intense. Fourteen lines (involving thirteen terms) were examined; the twelve lines in table 1 were found to have no structures.

Table 1. Cd II lines which have no structures

λ	Classification	λ	Classification	λ	Classification
4285	$8\,^2S_{\frac{1}{2}}$–$6\,^2P_{\frac{1}{2}}$	6759	$6\,^2P_{\frac{3}{2}}$–$6\,^2D_{\frac{3}{2}}$	7237	$6\,^2D_{\frac{3}{2}}$–$6\,^2F_{\frac{5}{2}}$
4412	$8\,^2S_{\frac{1}{2}}$–$6\,^2P_{\frac{3}{2}}$	5337	$5\,^2D_{\frac{3}{2}}$–$4\,^2F_{\frac{5}{2}}$	7284	$6\,^2D_{\frac{5}{2}}$–$6\,^2F_{\frac{7}{2}}$
6465	$6\,^2P_{\frac{1}{2}}$–$6\,^2D_{\frac{3}{2}}$	5378	$5\,^2D_{\frac{5}{2}}$–$4\,^2F_{\frac{7}{2}}$	6355	$4\,^2F_{\frac{5}{2}}$–$5\,^2G$
6726	$6\,^2P_{\frac{3}{2}}$–$6\,^2D_{\frac{5}{2}}$	5381	$5\,^2D_{\frac{5}{2}}$–$4\,^2F_{\frac{5}{2}}$	6360	$4\,^2F_{\frac{7}{2}}$–$5\,^2G$

---

[*] H. Schüler and J. E. Keyston, *Z. f. Phys.* **67**, 433 (1931).

[†] G. von Salis, *Ann. d. Phys.* **76**, 145 (1925).

[‡] H. Schüler, *Z. f. Phys.* **35**, 323 (1926).

The line λ 8067 ($6\,^2S_{\frac{1}{2}}$–$6\,^2P_{\frac{3}{2}}$) has a small structure comprising three components, an intense line (due to the unresolved even isotopes) bracketed by a much fainter doublet (with a separation $\delta\nu$ of 0·11 cm.$^{-1}$) which is ascribed to the odd isotopes. Since the absence of hyperfine structure in λ 6726 and λ 6759 proves that the level $6\,^2P_{\frac{3}{2}}$ is single, the measured structure in λ 8067 is that of the level $6\,^2S_{\frac{1}{2}}$. The intensity ratio and position of the faint doublet indicate that the odd isotopes have nuclear moments *I* equal to $\frac{1}{2}$ and that the hyperfine levels are inverted, in complete agreement with the analysis of Schüler and Keyston* for the visible triplet of Cd I. The related line λ 8530 ($6\,^2S_{\frac{1}{2}}$–$6\,^2P_{\frac{1}{2}}$), which is, however, too faint to be analysed completely, shows indications of the same structure as that obtained for λ 8067.

A number of new Cd II lines have been observed in the far red, and it has been found possible to classify three of these, namely

λ 8530·3 $6\,^2S_{\frac{1}{2}}$–$6\,^2P_{\frac{1}{2}}$
λ 8389·3 $7\,^2S_{\frac{1}{2}}$–$6\,^2P_{\frac{3}{2}}$
λ 7940·8 $7\,^2S_{\frac{1}{2}}$–$6\,^2P_{\frac{1}{2}}$.

The new doublet involving the $7\,^2S_{\frac{1}{2}}$ term, which had previously only been estimated from combinations in the extreme ultra-violet, fixes its value as 29,075·5 cm.$^{-1}$ These new lines were all too faint to be photographed successfully through the interferometer.

§ 3. DISCUSSION OF RESULTS

The measurement of a hyperfine separation in a ^2S-term of the odd isotopes renders possible a calculation of the magnetic moments associated with the nuclear spins.

a

It has recently been shown† that, for penetrating *s* electrons, the interval factor *a* may be expressed by the formula

$$a=\frac{8R\alpha^2 Z_i Z_0^2\,\kappa\left(\frac{1}{2};\,Z_i\right)}{3\,n_0^3}\cdot\frac{g(I)}{1838} \qquad \text{......(1)},$$

R, α, n_0
Z_0, Z_i
κ
$g(I)$

where R, α, n_0 are respectively Rydberg's constant, Sommerfeld's fine-structure constant, and the effective principal quantum number of the spectral term under consideration. Z_0 and Z_i are the effective nuclear charges experienced by the electron in the outer and inner parts, respectively, of the penetrating orbit. The function κ represents a relativity correction. On substitution of the appropriate numerical values (1) the $g(I)$ factor (the ratio of magnetic to mechanical moments) for the nuclei of the odd isotopes is found to be − 1·25 proton units. This is in very good agreement with the value − 1·33 calculated by Goudsmit‡ from the measurements§ on certain Cd I lines.

§ 4. ACKNOWLEDGMENT

The author wishes to express his thanks to Prof. A. Fowler, F.R.S., in whose laboratories the work was carried out, for his encouragement and help.

---

* *Loc. cit.* (i) $a=0{\cdot}11$, $n_0=2{\cdot}865$, $Z_0=2$, $Z_i=48$ and $\kappa=1{\cdot}28$.

† S. Goudsmit, *Phys. Rev.* **43**, 636 (1933).

‡ *Loc. cit.*

§ Schüler and Keyston, *loc. cit.*

535.338:546.27

THE INTENSITIES OF BANDS IN THE SPECTRUM OF BORON MONOXIDE

By A. ELLIOTT, Ph.D., D.Sc., Physics Department,
the University, Sheffield

Communicated by S. R. Milner, F.R.S., May 24, 1933.
Read July 7, 1933

ABSTRACT. The intensities of the α bands of BO have been measured by means of an arrangement which gives the intensity of a whole band. Precautions for the detection of systematic errors are described, and the probable error is estimated at about ± 5 per cent. The observed intensities have been compared with theoretical ones calculated from wave mechanics. Morse's wave function is found to give much better results than the harmonic wave function, but shows some discrepancy from theory for higher vibrational states. The theory using the harmonic wave function has been applied to some previous results for the intensities of the β bands of BO. Fair agreement is found for the most part, but a few bands show discrepancies greater than the experimental error. The intensity ratio of the $B^{11}O$ and $B^{10}O$ bands has been measured for two of the α bands. The result, 3·5:1, is in satisfactory agreement with a previous determination of the isotope intensity ratio in the β bands of BO, 3·66:1.

---

§ 1. INTRODUCTION

The probability that a transition will occur between two vibrational states of a molecule can be calculated by wave mechanics, provided that the molecular constants have been determined with sufficient accuracy. Such calculations have been carried out by Hutchisson (1, 2) and by Price (3). The accuracy of the calculated values depends on the accuracy of the simplifying assumptions which have to be made, and chiefly on the way in which the nuclear potential energy is assumed to depend on the internuclear distance.

As the experimental data available for testing these calculations are rather meagre, it seemed desirable to determine the intensities of a number of bands with as great an accuracy as possible. For such a purpose it is necessary that the bands measured should lie in a region for which accurately calibrated standard lamps are available, and that they should form a system in which overlapping of bands does not occur to a great extent. The α system of boron monoxide in the visible region seemed fairly suitable for the purpose, and was accordingly chosen. It is the object of this paper to describe the determination of the intensities of these bands, and to compare them with the theoretical intensities. The notation used here is that given by Jevons (5).

§ 2. EXPERIMENTAL

The method employed for the determination of relative intensities over a wide range of wave-lengths was substantially the same as that developed by Ornstein and his colleagues at Utrecht (6, 7), with the important addition that an arrangement recently described by Thompson (8) was employed in making the microphotometer record of the bands whose intensities were to be compared. This modification gives a record in which the ordinates are proportional not to plate density but to intensity multiplied by a factor which is constant over a range of about 50 A., i.e. over a complete band. The record is planimetered to give the area for a complete band. This, when multiplied by the appropriate factor, is a measure of the energy radiated as a result of the vibrational transition in question. Density marks were obtained by photographing the spectrum of a standard lamp through a step slit (9, 10) placed over the spectrograph slit, which was opened wide.

Calibration of standard lamps. Tungsten spiral-filament lamps, run at constant current from accumulators, were used; the colour temperatures of the lamps were determined by different methods. The first lamp used (6 V., 36 W.) had been calibrated at the Physical Laboratory of the University of Utrecht by means of an optical pyrometer (11) and was kindly given to the author by Prof. Ornstein. This lamp was used in method (*a*) to be described later. The lamp used in method (*b*) (12 V., 24 W., Siemens motor-headlight lamp) was calibrated in this laboratory by concentrating its radiation on the slit of a monochromator and observing the galvanometer deflections produced by a thermopile at two suitable wave-lengths ($\lambda\lambda$ 6563 and 5016). The ratio of these two deflections, after correction for the stray radiation scattered by the optical parts of the monochromator and for the different transmissive powers of this instrument at the two wave-lengths, enabled the colour temperature of the lamp to be calculated from Wien's law. A knowledge of the dispersion of the monochromator at each wave-length is also necessary for this calculation.

Stray light could have been eliminated by using a double monochromator, but as this was not available, screens transparent to the stray radiation (chiefly near infra-red) but opaque to the radiation which is to be measured were used as shutters (12). The difference between the galvanometer deflections with and without these shutters gives the deflection due to the radiation which is to be measured, provided that the shutter transmits all of the stray radiation. As shutters, the Ilford infra-red filter was used when measuring the radiation at λ 6563, and cellophane stained with commercial red ink for λ 5016. These screens transmitted only about 90 per cent of the stray radiation, and in consequence the difference of deflections with and without the shutter was larger than the deflection due to the radiation to be measured. At λ 6563 the percentage of stray radiation was so small as to render further correction unnecessary, but this was not the case at λ 5016. By observing the difference between the deflections obtained at this wave-length with a totally opaque shutter and with the red cellophane screen, the deflection due to the stray radiation transmitted by the screen was found. In similar fashion the deflections due to the stray radiation transmitted by two, three and four thicknesses of the same screen were found. By plotting these deflections against the thickness of screen used and then extrapolating to zero thickness, the deflection due to the stray radiation as it would have appeared with a screen transmitting 100 per cent was obtained. This result, together with the difference between the deflections with and without the single thickness of screen, enabled the deflection due to the radiation of wave-length λ 5016 to be calculated.

The quantity

$$\frac{\left\{\begin{array}{l}\text{(fraction of incident light transmitted by the}\\ \text{monochromator and condensing lens for } \lambda\ 5016)\end{array}\right\}}{\text{(fraction transmitted for } \lambda\ 6563)},$$

which may be referred to as the *relative transmission* for these wave-lengths, was determined in the following manner. The monochromator, lamp and condensing-lens were arranged in front of the slit of a spectrograph so that, when the thermopile was removed, the light in the region λ 5016 transmitted by the monochromator fell on the spectrograph slit. The monochromator was sufficiently far from the spectrograph to ensure that only the central part of the spectrograph prism was illuminated. The spectrograph was provided with a step slit. Density marks were photographed with this arrangement; then the monochromator was set to give the region λ 6563 and density marks at this wave-length were taken. The lamp was then placed between the monochromator and spectrograph slit, and density marks were again taken. The current through the lamp was of course kept constant, and all exposure times were the same. After the plate had been photometered in the ordinary way, the relative transmission for the two wave-lengths could be calculated. The method is an adaptation of that devised by Ornstein for the photographic method of calibrating standard lamps⁽⁷⁾.

The relative transmission of the monochromator is constant for two given wave-lengths, and hence has only to be measured once. When this determination has been carried out, it is a fairly simple matter to calibrate a lamp by observing the deflections due to the thermopile for two wave-lengths as described above, correcting them for the relative transmission and finally calculating the colour temperature of the lamp from Wien's law. The probable error in the determination of the ratio $E_{\lambda 5016} : E_{\lambda 6563}$ is about ± 1 per cent, giving a probable error of ± 10° in a colour temperature of 2930° K.

Intensity-measurements. The spectrum of boron monoxide was excited by allowing the vapour of boron trichloride to pass into streaming active nitrogen containing the necessary trace of oxygen⁽¹³, ¹⁴⁾. It is of importance in heterochromatic photometry that the exposure times be the same for density marks and for the spectrum in which the intensities are to be measured. Since the energy radiated by the standard lamp is very different for regions of different wave-length, some arrangement is necessary whereby a series of sets of density marks of varying intensity but constant exposure time may be made. Two methods were used.

Method (*a*). A single-prism Littrow spectrograph with glass optical system giving the whole of the visible spectrum on a 10-in. plate was employed for the measurement by this method. In this case the light from the standard lamp passed through the afterglow tube before falling on the spectrograph slit. The condensing lens, used to throw an image of the tube on to the slit, was removed when density marks were being taken. Several sets of density marks were taken with different lamp currents in order to get suitable densities over the wave-length range 4000 to 6000 Å.

When exposure for the BO bands and development had been completed the density marks were photometered at the wave-length corresponding to the most intense part of each band. Where the densities were suitable, two sets of density marks were photometered at the same wave-length so as to enable the relative intensities of the various sets, taken with different currents, to be determined at selected wave-lengths. The relative intensities at the same wave-lengths were then calculated from the colour temperature of the lamp filament, the filament being assumed to behave as a black body at a temperature equal to the colour temperature. The calculated relative intensities differ slightly from those determined experimentally as described above, because the colour emissivity of tungsten(15) is a function of the colour temperature. Correction factors could now be evaluated by which the energy functions E_λ calculated from the colour temperatures could be multiplied. The corrected values E_λ' when multiplied by $d\lambda\, ds$ (where s is the distance along the photographic plate) and slit-width w give the correct relative intensities of the light causing the density marks, even though different currents were used. Templates for the modified microphotometer were drawn with a needle point on smoked glass for each band. The ordinates were the degrees of blackening of the density marks and the abscissae were the widths w of the corresponding steps. The bands were photometered with the templates(8), and registrations whose ordinates were proportional to w and abscissae to s were obtained. The constants of proportionality depend only on the dimensions of the microphotometer, and similar factors, and are hence the same for all bands.

E_λ', s
w

The intensity of any part of a band is proportional to the ordinate on the registration at that point multiplied by $E_\lambda' . d\lambda\, ds$. The relative intensity of the whole band is given by multiplying the area under the registration by $E_\lambda' . d\lambda\, ds$. The relative intensities of thirteen bands were determined in this way from photographs taken on Ilford Special Rapid Panchromatic plates. The combined intensity due to $B^{11}O$ and $B^{10}O$ was measured in each case.

Method (*b*). The use of different lamp currents in order to secure suitable density marks over a wide range of wave-lengths leads to a rather complicated procedure for obtaining the values of E_λ', with the introduction of new errors of measurement at each step. In the method now to be described the lamp current remained constant. A magnesium-oxide screen was placed between the afterglow tube and condensing lens when density marks were to be taken; the plane of the screen was at about 45° to the axis of the condensing lens, which remained in position throughout. The standard lamp was placed so that its light fell normally on the white screen, whence it was diffusely reflected into the spectrograph. The intensity of illumination of the screen was altered by altering the distance d between

d

lamp and screen and was calculated from the inverse square law. This could be applied provided d was greater than 20 cm.; the limit is set by the dimensions of the filament and the accuracy sought, the error being 1 per cent.

The intensity of the light causing a density mark was now proportional to $E_\lambda(d\lambda/ds).(w/d^2)$ and was readily calculated. The arrangement had the further advantage that the whole of the prism could be utilized, provided that a large enough white surface was employed. This ensures that the path of the rays is the same for the light giving the density marks and that of the BO bands. The BO bands were photographed and the relative intensities were measured, the above method being used for taking density marks. The further procedure was the same as in method (*a*). In both methods it was necessary to take two photographs of the BO bands, one with a wide and the other with a narrow spectrograph slit, because of the great range of density which occurs on a single exposure. The correlation between the two sets was made from measurements on bands of medium density which were measurable on each plate. A Hilger E 2 quartz spectrograph was employed for the measurements made by method (*b*).

§ 3. COMPARISON OF RESULTS

The relative intensities of the BO bands measured by both methods are given in table 1, where columns (*a*) and (*b*) give the results from plates in which methods (*a*) and (*b*) respectively have been used.

In method (*a*) a standard lamp calibrated by means of an optical pyrometer was used, whereas in method (*b*) the lamp was calibrated by the thermopile method. Different step slits and spectrographs were employed, and the dispersion and prism absorption was widely different in the two cases. The ratio (*b*)/(*a*) is sufficiently constant for a single v'' progression to justify the belief that the average of the two v'' sets, after (*b*) has been divided by the mean ratio (*b*)/(*a*), represents the intensities of the α bands of BO to about ± 5 per cent. This average is given in column (*c*). It would seem that no serious systematic errors due to absorption by the spectrograph, faulty calibration of the standard lamp or step slit, or properties of the photographic plate, are present. Evidently higher accuracy could be attained by multiplicity of measurements.

The fact that the mean ratio (*b*)/(*a*) is different for bands coming from the initial states 0, 1 and 2 must be taken to mean that the excitation conditions were not the same in both cases, so that a different distribution of the initial states occurred. This is perhaps due to the fact that the active nitrogen was produced by the discharge from an induction coil in (*a*) and from a 16,000-volt transformer in (*b*).

§ 4. COMPARISON WITH THEORY

Hutchisson (1) has evaluated the integral

$$I = \int_{-\infty}^{+\infty} \psi_{e'v'}(r)\, \psi_{e''v''}(r)\, dr$$ I, e, v

assuming the nuclei to perform simple harmonic oscillations, and in his second paper (2) has extended the results to the case where the potential energy of the nuclei is expressed as a power series in a factor depending on the displacement of the nuclei from their equilibrium positions. He also suggests the use of the wave function derived from Morse's potential-energy function ⁽⁴⁾, and uses the method of graphical integration for determining I in this case for the molecule H_2*. Other wave functions in addition to these have been used by Price ⁽³⁾, who has also used them to calculate the values of I for H_2.

I have calculated the values of the above integral for the BO α bands, using Hutchisson's formulae for harmonic oscillations, and have also determined them by graphical integration, using Morse's wave function; the molecular constants have been taken from Jevons' *Report on Band Spectra of Diatomic Molecules* ⁽⁵⁾. As the calculations in the second method become rather laborious with increasing v, they have not been carried out for the 0–4 and 2–4 bands, which are of low intensity and hence are not so well determined experimentally as the others.

The intensity of an emission band is obtained by squaring I and multiplying by the fourth power of the frequency ν of the band and by the number of molecules in the upper state.

Table 1. α bands of BO

Band	λ	Intensity of band measured by method (a)	Intensity of band measured by method (b)	(b)/(a)	Average of (a) and of (b) divided by the mean value of (b)/(a)	Theoretical intensity $I^2\nu^4$ By harmonic wave function	Theoretical intensity $I^2\nu^4$ By Morse's wave function
0–0	4248	64	94	1·47	64	67	57
0–1	4613	153	208	1·36	147	126	165
0–2	5040	135	193	1·43	133	134	139
0–3	5548	102	164	1·61	107	102	103
0–4	6160	54	84	1·51	56	63	—
				Mean = 1·48			
1–0	4036	141	186	1·32	142	184	136
1–1	4363	160	204	1·27	159	168	187
1–2	4744	79	100	1·27	79	71	72
				Mean = 1·29			
2–0	3847	†	323	—	115	254	153
2–1	4143	72	174	2·4	67	60	76
2–2	—	—	—	—	Absent	0·07	13
2–3	4883	42	115	2·7	42	20	31
2–4	5351	51	165	3·2	55	22	—
				Mean = 2·8			

This last quantity is quite unknown, and so theory and experiment can only be compared for bands with a common upper level. The values of $I^2\nu^4$ in table 1 have been multiplied by a factor which is constant for each v'' progression and is chosen so as to give the best fit between theory and experiment. It will be seen that the calculations based on harmonic oscillations and on Morse's wave function both agree well with experiment for bands for which $v' = 0$ and 1, but when $v' = 2$ Morse's function gives very much better results. Even with Morse's function, however, there is a systematic difference between calculated and experimental intensities when $v' = 2$.

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* It should be pointed out that in Morse's paper, the denominator Γ $(s - 1)$ in the normalizing factor should be Γ $(s + 1)$.

† Absorbed strongly by the prism.

§ 5. ISOTOPE EFFECT IN α BANDS OF BO

It has been found possible to make a measurement of the intensity ratio of the bands of $B^{11}O$ and $B^{10}O$ for the 0–3 and 0–4 bands. As the isotopic bands are not completely separated the intensities of corresponding heads were measured, rather than the areas under the bands. The results for the intensity ratio of the isotopic bands are shown in table 2.

Table 2

	0–3 band	0–4 band	
Method (*a*)	3·8	3·5	Mean = 3·5
Method (*b*)	3·3	(Too faint)	

The correction (16, 17) to be made in the intensity ratio in order to obtain the relative abundance of the isotopes will be negligible here, as the bands both have v' equal to 0, and the energies of the $B^{11}O$ and $B^{10}O$ molecules differ very little for this level. The above result is in satisfactory agreement, for the relative abundance of the isotopes, with that for the β bands of BO, viz. 3·63.

§ 6. THE β BANDS OF BO

In a previous paper (9, 16) the writer has described the measurement of the intensities of the band heads in the β system of BO, and the determination (based on certain assumptions) of the probabilities of transition p between a number of vibrational states. This probability p is obtained by dividing the measured intensity of a band by ν^4 and by a factor proportional to the concentration of emitters in the particular v' state concerned. It should consequently be proportional to the square of the integral I given by Hutchisson, and the constant of proportionality should be the same for all bands in the system, irrespective of v'. Although no great precision was attempted in the measurement of the β bands* it seemed worth while to calculate I for a number of these bands, Hutchisson's calculations for harmonic oscillations being used. This was accordingly done; table 3 gives the value of I^2 below the corresponding value of p, and the values of I^2 have all been divided by a constant factor to make them comparable with p. Quite good agreement is to be found when $v' = 0$, and for the other progressions the agreement is fair. A notable discrepancy occurs for the 1–3 and 1–4 bands, and is too large to be accounted for by experimental error. It is very satisfactory to note that there is no systematic discrepancy between I^2 and p as v' alters. This may be taken to mean that the procedure (16) adopted for obtaining the number of emitters in the various v' states gives a substantially correct result. It may be mentioned that this point cannot readily be tested on the α bands, as it is necessary, in order to obtain the distribution of emitters in the initial state, that the intensities of all the bands in the system be measured. There is reason to believe that a considerable part of the α system lies in the infra-red beyond the range of the photographic plates used.

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* The error is probably as high as ± 20 per cent in certain bands, though the error in the intensity ratios of neighbouring bands in a progression will be much smaller than this.

It would appear that in the β bands, as in the α bands of BO, the theory based on harmonic oscillations gives results which are in many cases in agreement with experiment, but that in certain bands the discrepancy between theory and experiment is greater than the experimental error. No doubt better agreement would be obtained if the theoretical intensities were calculated from Morse's energy function, but the accuracy of the measurements is hardly high enough to warrant the labour involved in these calculations.

Table 3. Transition probabilities in β bands of BO

v'' v'	0	1	2	3	4	5	6	7	8	
0	24 20	25 25	29 21	13 14	8 8					p I^2
1	25 37	6 10	— 0·6	21 7	22 56	15 12	11			p I^2
2	30 28	— 6·5	13 14	5 5	2 0·01	19 3	20	10		p I^2
3	17 10	16 29	— 0·6	4 6	20 9	— 0·9	8	21	14	p I^2

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A HIGH-TEMPERATURE X-RAY CAMERA FOR PRECISION MEASUREMENTS

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ABSTRACT. A high-temperature X-ray camera has been designed for taking powder photographs which show well-resolved K_α doublets at high angles of reflection, enabling an accurate estimate of line position to be made. The temperature of the specimen was found in the following way. Photographs of silver, from which the lattice dimensions were calculated, were taken with varying heating currents. The coefficient of expansion of silver was used to convert lattice-spacing measurements to degrees centigrade. A curve relating the watts in the furnaces to the temperature of the specimens was thus obtained. A test of the reliability of the method was carried out by means of X-ray experiments on the expansion of quartz and the conclusion reached was that the assumption regarding the expansion of silver was valid. The temperature scale thus calculated was found to be correct to within 5° C.

§ 1. INTRODUCTION

Changes which may be found in the crystal structure of metals and alloys as the result of varying heat treatment do not always correspond to true equilibrium conditions. The time taken for an alloy structure to attain equilibrium varies enormously with the temperature. Equilibrium between neighbouring atoms may be established in a fraction of a second at temperatures just below the melting point, while in the case of high-melting alloys the attainment of equilibrium at room temperature is virtually impossible. Over a particular range of temperatures gradual changes take place during a finite period, and eventually equilibrium may be attained. Further transformation may be arrested if an alloy is suddenly cooled, and in some cases it may be possible to preserve the form characteristic of the temperature from which the alloy was quenched. At higher temperatures equilibrium is attained almost instantaneously, and here the structure cannot be retained by quenching. The use of a high-temperature X-ray camera is essential for temperatures above those at which quenching is applicable.

The pioneer X-ray investigations of alloy structures at high temperatures were those carried out by Westgren and Phragmen*. The camera was of the Debye-Scherrer type, the substance under examination being in the form of a wire which was heated to the required temperature by passing a current through the wire. Later workers† have used similar methods with cameras of slightly modified design. Becker has made measurements on the expansion of substances attached to a heated filament wire. In Cohn's experiments the substance was heated with a surrounding coil of wire.

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* A. Westgren, *J. Iron and Steel Inst.* **103**, 303 (1921); A. Westgren, A. E. Lindh, G. Phragmen, *Z. f. Phys. Chem.* **98**, 181 (1921); **102**, 1 (1922).

† K. Becker, *Z. f. Phys.* **40**, 37 (1926); W. M. Cohn, *Z. f. Phys.* **50**, 123 (1928).

In designing the present high-temperature camera the aim was to preserve, as far as possible, the essentials of the powder-camera which the author has used and found highly satisfactory* and to introduce such heating and cooling elements as would not interfere with the photographing of the specimen.

§ 2. DESCRIPTION OF CAMERA

The camera is constructed in two main pieces, camera proper and detachable cover. The camera itself consists of two circular flanged discs held rigidly, in the form of a cylinder, by a block which contains the slit system for the entrance of the primary X-ray beam. The dimensions are: diameter of flanges, 9 cm.; total length of slit system, which is cylindrical, 2·5 cm.; entrance, diameter 3 mm.; exit, diameter 4 mm.; step with 2 mm. hole, 5 mm. from exit. The apex of the sides of the **V**-shaped block, figure 2 (*b*), is a vertical line 5 mm. in front of the axis of the camera. The knife edges, which give the sharp shadows on the X-ray film, are part of a plate which is permanently fixed on the central block, figure 2 (*b*), the knife edges being on the circumference of the camera flanges around which the X-ray film is wrapped. A cover plate is fixed over the knife edges, sufficient room being allowed for the film and paper covering to slide between it and them. An overlap of several mm. is essential for good contact of the film against the knife edges.

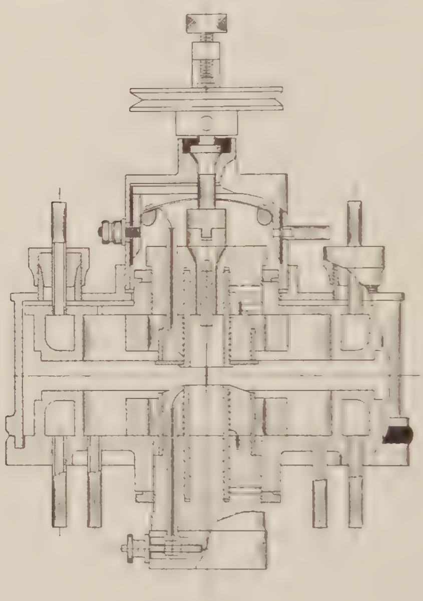

Figure 1.

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* A. J. Bradley and A. H. Jay, *Proc. Phys. Soc.* **44**, 563 (1932). (Communicated to *Z. f. Krist.*)

The heating is provided by two small wire-wound furnaces, figures 3 (*a*), (*b*) whose position within the camera can be adjusted by the screw fitting and maintained by locking-nuts. The resistance wire* is wound around silica tubes which are held in steel blocks by small screws. In both furnaces one end of the wire is fixed to the block and this makes a common join with the rest of the camera when the furnaces are in position. Because the body of the camera forms part of the electrical circuit, the adjustable stand is insulated from the platform of the X-ray tube. The other end of the wire is in each case insulated by silica tubing and led to a terminal. For the lower furnace the terminal is fixed in a bakelite block on the exterior of the camera. In the upper furnace the wire is led to an insulated spring terminal of which details may be seen from figures 1, 3 (*b*). The shape of the block ends is such that only a small bend of the spring is needed for good contact with the insulated collar of the cover when the latter is in position. The necessary lagging for the furnaces is a mixture of equal parts of alumina powder and alundum cement. This has the advantage over alundum cement alone in that it avoids cracking due to excess shrinkage with repeated heating. An outer metal sheath provides the necessary strength.

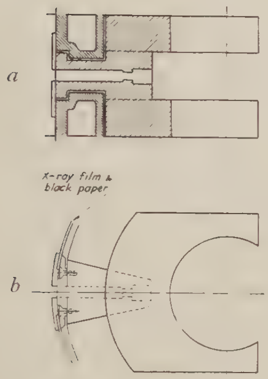

Figure 2*a*, *b*.

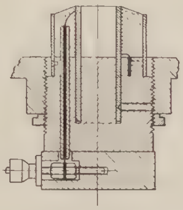

Figure 3*a*.

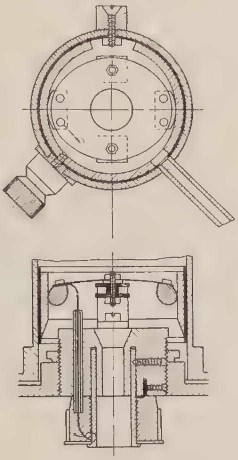

Figure 3*b*.

To maintain the X-ray film at room temperature, cooling is effected with two circular water-jackets round the inside of the camera flanges, each with its own inlet and outlet pipes, see figure 1. To ensure that the central strip of the X-ray film which is not in contact with the cold flanges shall not be heated by heat radiation, a screen is placed between the X-ray film and the specimen. This screen of light-proof black paper, strengthened by celluloid film, is wrapped round the small step as shown in figure 4 and held in position by rubber bands. The object of separating the inner black paper from the X-ray film is to allow the film to make contact with the knife edges, and thus enable a sharp shadow to be cast on the film by the general

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* Resistance 4 ohms/yard, $\frac{3}{4}$ yard on each furnace.

X-ray scattering from the specimen. Experiments with furnace temperatures up to 1000° C. have shown that the arrangement works satisfactorily.

At the bottom of the camera are three tubes for gas inlet. Two are placed on the inner side of the water-jacket, 2 cm. on either side of the furnaces and opposite each other. The third tube allows the gas to enter the space between the film and the outside cover. Small holes in the top camera flange allow the gas to escape. When the camera is required for vacuum work these gas tubes may be used for evacuating it.

The steel specimen-holder rotates in the upper furnace block, figures 1, 5, in a ground cone joint which is made air-tight to prevent a disturbing stream of cold gas from flowing through the furnace. A silica rod, $\frac{3}{4}$ mm. in diameter, is held vertically in the holder, and on this the specimen is mounted and rotated in the path of the X-ray beam.

The details of the cover may be seen from figures 1 and 3 (*b*). The small upper dome encases the rotation gear and spring terminal, and serves to collect the gas which is then led away. Inside this dome is the insulated collar to which reference has been made. The collar is connected to an outside terminal. A pulley and shaft enable the specimen-holder to be rotated. The bearings of the pulley-shaft and cover form a ground joint which is effectively sealed with vacuum oil* contained in a small cup. The exit tube for the main X-ray beam is lead lined and the outer seal is made with a glass cover-slip. Glass is used in preference to metal foil since it gives no back-line reflection. To keep the cover light-tight a black paper cap is put over the end of the tube. A thick lead covering is placed round the front of the camera as a shield against penetrating radiation from the X-ray tube. The opening in the cover for the X-rays is sealed with thin aluminium foil. On the top of the cover are small tubes through which the water-pipes can pass. When the cover is in position the seal is made with rubber bungs held by screw caps. The cover makes a light-tight joint with the lower flange.

The two circular flanges are permanently united by the central block. The furnace blocks are screwed into a position where the X-ray beam from the slit system passes between the ends of the furnaces without striking them; with the given camera dimensions the separation of the two furnaces is about 4 or 5 mm. The furnace blocks are then clamped in position by locking-nuts. The protecting screen, consisting of the black paper and celluloid film, is wrapped round the camera on the steps as shown in figure 4 and is kept in position by rubber bands. Slots in the V-shaped central block allow an overlap which provides for the possibility of any small shrinkage of the paper and film due to heating.

When the specimen is being heated in a gas atmosphere the joint between the cover and the lower flange of the camera is sealed with plasticene. The flow of gas through the camera need not exceed 3 cm^3/min. Where it is necessary to carry out the experiment *in vacuo*, the joint referred to above is made air-tight in the following way. A rubber ring is placed between the rim of the cover and the lower flange, and compressed by clamps. The camera is continuously evacuated with a rotary oil pump.

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* Apiezon J, made by Shell-Mex Ltd. under Metro-Vickers patents.

§ 3. METHOD OF HEATING THE SPECIMEN

One of the advantages possessed by a high-temperature camera in which the specimen is heated with a surrounding furnace instead of a filament wire through its centre is that a much more uniform temperature can be obtained throughout the scattering mass. With central-wire heating the specimen acts as a lagging for the hot wire and there is an appreciable temperature-gradient from the centre outwards. With different specimens this will be an unknown and variable factor. In the case of materials such as refractories, where the absorption is comparatively low, the scattering takes place throughout the whole mass. A photograph of a specimen heated in this way would show a lack of definiteness in the lines, due to the superposition of slightly varying patterns. Moreover the temperature of the outer layers, which effect most of the scattering, is not that of the filament, the lattice spacing of which cannot be taken as a sure guide to the temperature of the specimen.

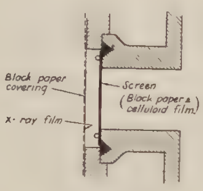


Figure 4.

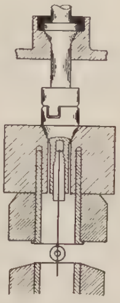


Figure 5.

§ 4. EXPERIMENTAL

The choice of a substance for mounting the powder grains as a cylindrical specimen presents some difficulty. Among the substances suggested by previous workers are water-glass, gypsum, zirconia and porcelain products. The list is somewhat limited and the final choice depends largely on the material to be investigated. This binding substance must not add unnecessary X-ray spectra or cause contamination of the specimen by chemical reaction. Water-glass as an adhesive appears to have been favoured, but the possibility of reaction with the powdered specimen must not be overlooked.

The following method of mounting the powder has proved very satisfactory. The thin silica rod, $\frac{3}{4}$ mm. in diameter, which is used in supporting the specimen is ground to $\frac{1}{3}$ mm. near its lower end. At the other end the rod is fused to make a bead by which it is supported in the holder. The silica rod is thus held in a vertical position and along the axis of rotation of the holder. Good centering, giving a minimum of lateral movement during rotation, is essential and is obtained by a good fit of the rod in the holder chuck. The powder is fine enough to pass through a 250-mesh sieve, which gives particles less than 0·05 mm.; it is mixed to a paste with a very dilute solution of gum arabic and is applied as a coating around the narrow part of the rod. The gum acts as an adhesive until it chars, at a temperature which may be of the order of 200° C. For higher temperatures the gum acts as a temporary binding, and subsequently the grains hold together by sintering. The amount of carbon formed is exceedingly small, and in our experiments it was burnt away in an atmosphere of hydrogen. Photographs of specimens at temperatures of 800° C. have been taken and it has been found that the powder has adhered.

Procedure. The specimen powder is mixed to a paste with a dilute solution of gum, is applied as a coating over the narrow part of the silica rod as described above, and is afterwards placed in position in the camera. The specimen-holder makes a gas-tight joint with the steel block.

The camera is loaded in a photographic dark room, where the cover is put in position and the seal between the water-pipes and cover is made by means of rubber bungs held by screw caps. Finally the seal between the cover and the lower flange is made.

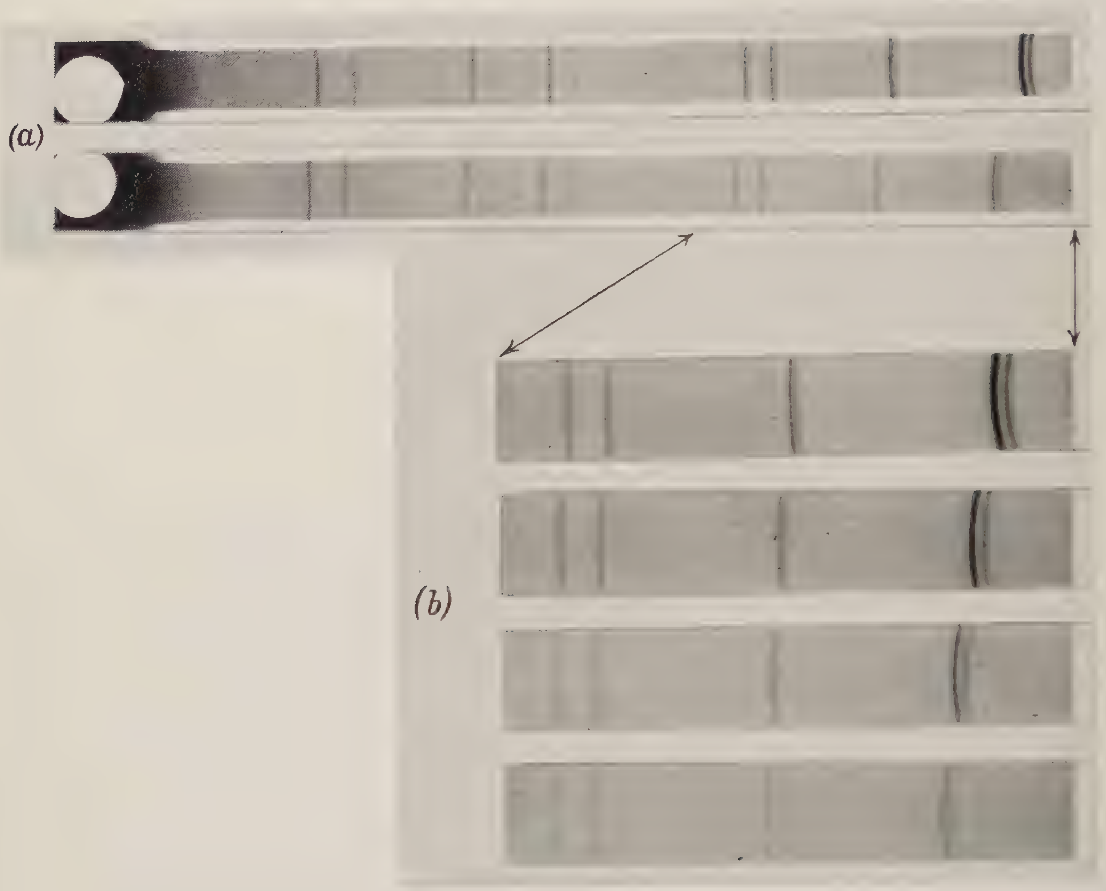
The procedure for adjusting the camera in front of the X-ray tube is similar to that of our standard powder-camera. One preliminary adjustment of the camera and stand is sufficient for a series of experiments as the stand is provided with pivot points.

When the camera has been placed in position the water and gas supplies and electrical leads are connected. The water is supplied from a constant head, the flow being 350 $cm^3/min.$ through each flange. The heating current through the furnaces is raised to the required value and maintained there for a definite time. The specimen is rotated by the pulley drive during the whole of the exposure, a complete rotation being made every few seconds. The exposure-time depends on the substance under investigation and on the radiation employed. With a tube current of 30 mA. at 52 kV. the times range from 5 minutes, for Al, Ni, Cu, Fe and Ag, to $\frac{3}{4}$ of an hour for rock salt and silicates, for characteristic K_α radiation, β radiation being eliminated with a screen.

§ 5. RESULTS. THE RELATION OF ENERGY INPUT INTO THE FURNACES AND THE TEMPERATURE OF THE SPECIMEN

One of the aims of the camera design was to make it possible to reproduce temperature conditions by using a known heating current. For this purpose the furnaces are strongly made, and throughout the whole series of experiments the flow of water through each water-jacket is maintained constant. The small gas stream through the camera is not sufficient to disturb the temperature conditions, being away from the furnaces. The outside of the camera is quite cold, so that radiation losses from the camera are negligible.

To find a relation between energy-input and the temperature of the specimen the following method is employed. An X-ray photograph of a standard material, pure silver, is taken for a given heating current in the furnaces. This procedure is repeated under varying heating conditions. From measurement of the position of the K_α doublets at high angles of reflection an accurate value of the lattice spacing



Powder photographs of pure silver.

corresponding to a measured heating current is obtained by means of the $\cos^2 \theta$ method* of extrapolation.

From the results a graphical relation between energy input and lattice spacing is obtained, figure 6. We now make the assumption that the coefficient of thermal expansion of silver as obtained by direct measurement is the same as that for the expansion of the atomic lattice†. In a later paper‡ we shall show from measurements on the expansion of quartz that this assumption is valid. It might be possible that both silver and quartz have the same difference between X-ray and optical dimensions, but it is unlikely that they would vary with temperature in exactly the same way along both the axes of quartz. Such a supposition is certainly not supported by the results for quartz, where the α-β change point is found at 579° C., for this agrees, within the limits of experimental error, with the acknowledged temperature, 575° C. The coefficient of expansion of silver as found by Scheel§ was used to convert lattice spacing to degrees centigrade. A temperature scale is included in the figure.

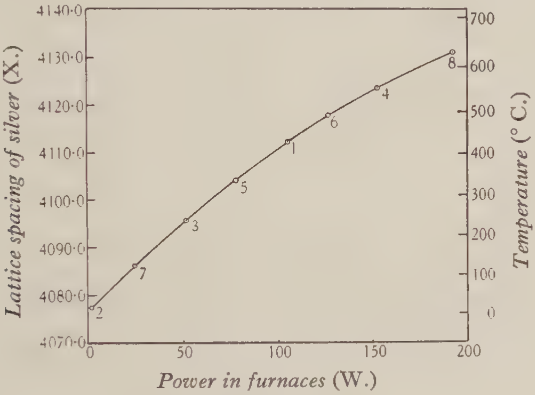


Figure 6.

To test the consistency of the observations the conditions were chosen at random, the order in which the photographs were taken not being the order of increasing temperature. This gives an opportunity of detecting any systematic change in the furnaces after heating. The same specimen was used throughout the experiments and no indication is given by the graph, figure 6, of any departure from a smooth curve, such as might be expected if a factor depending on time was present. The numbers opposite the points on the graph denote the order of the experiment, the order with increasing temperature being 2, 7, 3, 5, 1, 6, 4 and 8. Thus the method appears to be reliable, definite temperature conditions being obtained from definite heating currents. Granted that temperature can be calculated from spacing measurements, the temperatures are probably accurate to 3° C.

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* A. J. Bradley and A. H. Jay, *Proc. Phys. Soc.* **44**, 563 (1932).

† Compare F. Zwicky, *Proc. Nat. Acad.* **15**, 253; 816 (1929), *Hel. Phys. Acta*, **3**, 269 (1930), and A. Goetz and R. C. Hergenrother, *Phys. Rev.* **40**, 137, 643 (1932).

‡ A. H. Jay, Communicated to the Royal Society, May 6, 1933.

§ K. Scheel, *Z. f. Phys.* **5**, 167 (1921).

Below the complete photographs of silver at (*a*) in the plate is shown at (*b*) a series of enlargements of that part of the film which is measured for the determination of the lattice spacing. The photographs show the inward displacement of the lines for rise of temperature. In addition, the effect of temperature in reducing the intensity of the high-order reflections can be seen from a comparison of the end lines in succeeding photographs.

It might be thought surprising that a thermocouple has not been used to measure the temperature. There are several reasons for the omission of a thermocouple in the present case. One is the difficulty of placing the couple at the exact position occupied by the specimen. Another is that since the couple is heated by radiation, the conductivity of the leads would result in its having a lower temperature than the specimen. The difference would be a doubtful quantity and variable with the temperature. It might be possible to place the thermocouple at some convenient point of the furnace and use it as a temperature-pointer when once the camera had been standardized. However, since it was found that there was such a simple relation between energy-input and the temperature of the specimen it was considered unnecessary to add a further complication by the addition of a thermocouple.

§ 6. ACKNOWLEDGMENTS

I wish to thank Messrs Kay and Reynolds for their assistance in the construction of the camera which was made in the Laboratory. I am indebted to Dr A. J. Bradley for his helpful discussions and also to Prof. W. L. Bragg, F.R.S., for his encouraging interest in the work, which was carried out in the Physical Laboratory of the University of Manchester. I also thank Mr A. P. M. Fleming, C.B.E., Director-Manager, Research and Education Department of the Metropolitan-Vickers Electrical Co. Ltd., for his interest and for permission to publish the work.

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CONDITIONS FOR SECURING ACCURACY IN SPECTROPHOTOMETRY

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ABSTRACT. The paper sets forth basic principles in the design and use of spectrophotometers. §§ 1–4 deal with absorption spectrophotometry in its application to chemistry and includes consideration of the various methods in use, viz. photographic, visual, and photoelectric. The optimum conditions for each method are stated and the three types of method are compared. § 5 is concerned with spectrophotometry as a means of describing a radiation, as for instance in connexion with colorimetry, the comparison of light sources and quantitative spectrum analysis.

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§ 1. THE PURPOSES OF SPECTROPHOTOMETRY

The radiations which enable one to discriminate between substances (as in most of the applications in chemistry, such as the estimation of dyes and other absorbing substances) are those which are most absorbed. The radiations which chiefly determine a colour are, on the contrary, those which are most copiously present.

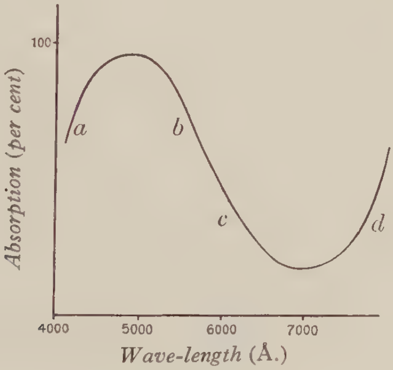

Figure 1.

For example, if a substance has the absorption curve shown in figure 1, the portion of the curve of outstanding importance as distinguishing the absorbing substance is that marked from *a* to *b*: yet in producing colour, that portion marked *c* to *d* is of paramount importance. In this paper we consider separately these two purposes to which spectrophotometry may be applied.

The measurement of absorption. The function of absorption that is appropriate

ϵ for this purpose is the Roscoe-Bunsen extinction coefficient ϵ, which for any substance and wave-length is, according to Beer's law, proportional to the concentration of the absorbing material. Thus the proportional accuracy with which a substance can be characterized by its absorption follows that with which ϵ can be determined, $\Delta\epsilon$ and is represented by $\Delta\epsilon/\epsilon$ where $\Delta\epsilon$ is the smallest measurable increment of ϵ.

The measurement of a radiation. A radiation, on the other hand, is best characterized by its intensity, and the proportional error of its measurement is represented by $\Delta I/I$, where ΔI is the smallest measurable increment of I, the intensity of the radiation.

We shall examine separately these two classes of measurement, which require differing modes of attack if the highest accuracy is to be attained.

§ 2. THE CHARACTERIZATION OF A SUBSTANCE BY ITS ABSORPTION

The type of problem we have in mind here is well illustrated by the application of absorption spectrophotometry to chemical analysis. It will be useful to give examples of this, and then to continue with a few considerations which are common to all methods of spectrophotometry.

The algebraical expression of Beer's law, and its use in the analysis of absorbing solutions. It has long been known how, if one has a mixture of two absorbing substances whose absorptions are known for two wave-lengths, to obtain by the application of Beer's law the proportions in which the two substances are present in the mixture*. What follows is the procedure described in the reference, with a very simple corollary thereto.

The following nomenclature will be used:

$\alpha_0, \alpha_1, \beta_0, \beta_1$ Let $\alpha_0, \alpha_1, \beta_0, \beta_1$ be the extinction coefficients of mixtures α, β, for wave-lengths
λ_0, λ_1 λ_0, λ_1.

a_0, a_1, b_0, b_1 Let a_0, a_1, b_0, b_1 be the extinction coefficients of the component solutions A, B... for wave-lengths λ_0, λ_1.

$m_\alpha, m_\beta, n_\alpha, n_\beta$ Let m_α, m_β, n_α, n_β, be the concentrations of solutions A, B, in mixtures α, β, in terms of that of the standard solutions.

Example 1. A mixture of two solutions A and B whose absorption curves are known. It is required to find the concentration of each solution in the mixture.

By Beer's law:

$$\alpha_0 = m_\alpha a_0 + n_\alpha b_0,$$

$$\alpha_1 = m_\alpha a_1 + n_\alpha b_1,$$

from which

$$m_\alpha = \frac{\alpha_0 b_1 - \alpha_1 b_0}{a_0 b_1 - a_1 b_0},$$

and

$$n_\alpha = \frac{\alpha_0 a_1 - \alpha_1 a_0}{b_0 a_1 - b_1 a_0}.$$

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* See K. Vierordt, *Die Anwendung des Spektralapparates zur Photometrie der Absorptionsspectren und zur quantitativen chemischen Analyse* (Tübingen, 1873); on p. 51 Vierordt describes how he checked the method on a mixture of potassium permanganate and potassium dichromate.

See also K. Vierordt, *Die quantitative Spektralanalyse in ihrer Anwendung auf Physiologie, Physik, Chemie, und Technologie* (Tübingen, 1876), for an account of work on absorption spectrophotometry.

It will be observed that to find m_α and n_α we only need to know the extinction coefficients of solutions A and B and of the mixture for two wave-lengths. The wave-lengths need to be selected with consideration in each specific problem, in order that high accuracy may be attained in the determination of m_α and n_α.

Example 2. Mixtures of two substances A and B, the absorption curve of only one of these being known.

If one makes observations on a number of such mixtures (α, β, γ, etc.) in which the concentrations of the substances A and B are different, then one is able to determine the ratio of the concentrations of the unknown substance B in any two of these mixtures.

As in example 1, we can write down two equations for a mixture:

$$\alpha_0 = m_\alpha a_0 + n_\alpha b_0,$$
$$\alpha_1 = m_\alpha a_1 + n_\alpha b_1.$$

Similarly we have two equations for another mixture:

$$\beta_0 = m_\beta a_0 + n_\beta b_0,$$
$$\beta_1 = m_\beta a_1 + n_\beta b_1.$$

In these four equations, there are six unknowns, m_α, n_α, m_β, n_β, b_0, b_1, so that a complete solution is impossible.

Nevertheless, the form of the equations enables one to obtain one piece of useful information, namely, the ratio

$$\frac{n_\alpha}{n_\beta} = \frac{\alpha_0 - (a_0/a_1).\alpha_1}{\beta_0 - (a_0/a_1).\beta_1}.$$

As an example of a problem to which this would be applicable, we may mention ethocaine. This substance is stated to hydrolyse slowly in aqueous solution. A direct application of the above test would therefore allow one to investigate the progress of the decomposition.

As a concrete case, we may consider the curves of figure 2. Curves 1, 2 represent the absorption of two mixtures of an unknown substance, in different proportions, with benzoic acid. Curve 3 represents the absorption of a solution containing benzoic acid only.

In the case in which $\lambda_0 = 2500$, $\lambda_1 = 2800$, we can obtain from the curves:

$$\alpha_0 = 0{\cdot}525, \quad \alpha_1 = 1{\cdot}14.$$
$$\beta_0 = 0{\cdot}395, \quad \beta_1 = 1{\cdot}62.$$
$$a_0 = 0{\cdot}565, \quad a_1 = 0{\cdot}59.$$

Substituting in the expression obtained we find that

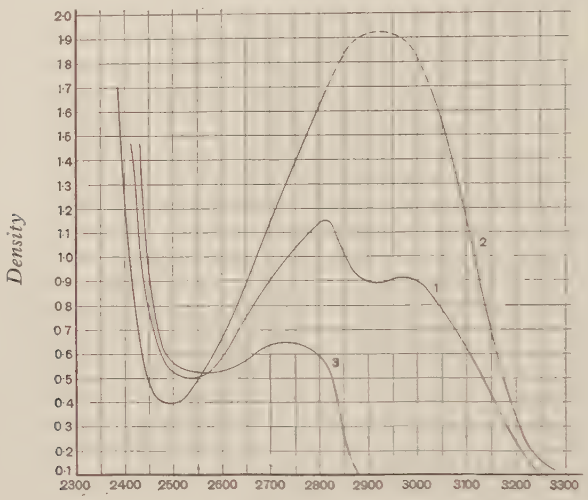
$$\frac{n_\alpha}{n_\beta} = \frac{0{\cdot}525 - 1{\cdot}09}{0{\cdot}395 - 1{\cdot}55} = \frac{0{\cdot}565}{1{\cdot}155} = 0{\cdot}489.$$

The choice of the most suitable pair of wave-lengths for analysis. The accuracy attained in a method of analysis such as the above will also depend considerably on the choice of the wave-lengths. The question is not susceptible of a finite method of analysis, because one does not know the equations to the curves obtained. The best way is to use the method of trial and error, in conjunction with careful inspection of the curves and of the form of the equation giving the required solution.

As an example, one can consider the calculation above based on the formula

$$\frac{n_\alpha}{n_\beta} = \frac{\alpha_0 - (a_0/a_1).\alpha_1}{\beta_0 - (a_0/a_1).\beta_1}.$$

Suppose that the "probable numerical error" in each of the above quantities on the right-hand side has the same constant value irrespective of the magnitude of the quantities themselves. Then the probable numerical error in the value of the numerator is constant, so that to make the fractional error small the numerator should have as large a value as possible. Thus we should endeavour to choose values so that *the difference between the two numerator terms is as large as possible*, and also that between the two denominator terms.



Wave-length (Å.)

Figure 2.

Supposing we have $\alpha_0 > \alpha_1$, then $a_0\ a_1$ should be small; similarly, one requires that β_0/β_1 should also be as great as possible. Thus in a general way the suitable conditions are that $\alpha_0\ \alpha_1$, $\beta_0\ \beta_1$ must be as large as possible, with $a_0\ a_1$ as small as possible, or alternatively $\alpha_0\ \alpha_1$, $\beta_0\ \beta_1$ must be as small as possible with $a_0\ a_1$ as large as possible. From figure 2 these conditions are apparently best attained at $\lambda_0 = 2500$, $\lambda_1 = 2800$.

Alternatively the wave-lengths might be chosen by direct consideration of the condition set out above in italics. Thus, in the numerator of the calculation given as an example in example 2 above, the difference 0·565 is comparable with both 0·525 and 1·09. If, instead, wave-lengths 2500 and 2600 had been chosen, the corresponding figures would have been 0·525 − 0·64, or − 0·115, from curves of figure 2.

In the case actually calculated an error of 0·01 (see p. 650) in density would cause an error of about 2 per cent in the final answer, but with the second pair of wave-lengths an error of 0·01 in density would involve an error of as much as 10 per cent in the final answer.

The dispersion system. The points of an absorption curve where great accuracy is required (such as those chosen above) will often be on the slope of the curve, so that if the measurements made on different occasions are to correspond we must ensure that the wave-lengths used are always the same, within the limit of the following condition: *The error of density due to error in wave-length setting should be less than the error in the density as measured.* Thus, in making use of a continuous spectrum, an instrument of sufficiently high dispersion is needed; and the steeper the curve at the point under consideration, the greater the dispersion desirable.

To take an illustration: in the use of a visual spectrophotometer for determining small percentages of carboxyhaemoglobin in blood, useful accuracy can only be attained by making measurements near the two wave-lengths 5600 and 5770 Å. At the latter wave-length the spectrophotometric curve of carboxyhaemoglobin is so steep that very narrow slits must be used in order to get a spectrophotometric measurement which has useful meaning.

In consequence of small brightness at large densities, it may sometimes be desirable to increase the width of the collimator slit. Where the source has a line spectrum this slit can be opened considerably without sacrifice of purity of spectrum, for one can then use the same lines on each occasion so that the above condition as to wave-length setting is automatically fulfilled. With a continuous-spectrum source, however, one must effect a compromise between the best light-intensity and the desired purity of spectrum.

Light-source. In all methods of visual or photographic spectrophotometry in which the intensities of two closely adjacent spectra are compared, it is essential that the relative intensities of such spectra should depend only on the absorption of the absorbing substance which is placed in one beam, and the deliberate variation of intensity which is imposed on the other beam.

An essential condition for accurate spectrophotometry with the types of instrument which the present authors consider best for this work is that the rays which have passed through the middle of the slit (which corresponds with the point of the spectrum where comparisons of intensity are made) should pass through the spectrograph *without any vignetting*. That this condition is fulfilled can be tested by looking through the spectroscope after removal of the eyepiece at an image of the light-source, the whole of which should be seen within the aperture of the apparatus. A corresponding test can be made with a spectrograph.

The light reaches the spectroscopic apparatus through two separate optical paths, so that if the photometric part of the apparatus is out of adjustment two such images of the light-source, not quite coincident, may be seen. If, then, the light-source is out of centre one of the images may be vignetted even if the other is not, or one may be vignetted to a greater extent than the other. In this way, a difference may be introduced between the effective relative intensities of the two beams.

It is thus necessary to have the light-source *small enough to be included within the aperture of the spectroscope or spectrograph*. This is even more necessary when, as usually occurs in ultra-violet spectrophotometry, there is very considerable chromatic aberration in the lens systems of photometer and spectrograph. But in order to obtain high accuracy large densities must be used, for these are needed for high accuracy in the characterization of substances by means of their absorption; and consequently as strong an intensity of illumination as possible must be obtained. But only by filling the aperture with light can the intrinsic brightness of the light-source be fully utilized. Thus the condition stated above in italics is antagonistic to the condition for high accuracy in the measurement of extinction coefficients.

Were it possible to obtain a light-source of great intrinsic brightness which at the same time was entirely uniform in radiation per element of its radiating area, the best course to adopt would be to produce an image which would entirely fill the aperture at all wave-lengths. Such a light-source, however, only exists in the form of an enclosed chamber within which the light-source is situated, and with such a means of illumination the intrinsic brightness is insufficient to give accuracy in those measurements of absorption with which we are now concerned. Thus the condition to be aimed at is that the trace of the light-source should pass through the spectroscope without being vignetted, but with no more margin than is needed to provide for any excentricity that may be accidentally present.

Similar considerations are applicable in the objective methods of spectrophotometry using photo-electric cells and the like.

The accuracy with which cells for spectrophotometry should be made

Visual and photographic measurements. The accuracy of the cells should be such that the departure from the nominal length of liquid should cause no measurable error. The least error in a density-measurement likely to be achieved in practice is probably equal to ± 0·005 in the density-reading. At a density of, say, 1·5, this is equal to 0·3 per cent. Thus the error should not be greater, for the best work than, say, one-half of this, i.e. 0·15 per cent.

Photo-electric measurements. As will be shown on p. 653, the best density to measure with a photo-electric null method is 0·4343. In measuring such a density a very much greater *sensitiveness* of measurement can be attained than with either visual or photographic methods. Those who have made careful comparisons of such measurements with those attained by the most careful visual spectrophotometry are agreed that absolute measurements of density made by photo-electric null methods do not yet yield an accuracy in accordance with this degree of sensitiveness*; but however this may be it will be seen that, in measuring a density of 0·4343, cells with an accuracy of ± 0·15 per cent will not be responsible for an error of extinction coefficient greater than that which corresponds with an error of ± 0·00055 in the density-measurement. Not till an accuracy of density-measurement approaching this is expected will any higher accuracy of cell become necessary.

The accuracy required in setting up the cell. Unless the cell is set every time perpendicular to the light beam, the length of light-path through the absorbing medium will vary. Suppose that the normal to the cell makes a small angle i with the incident light beam. If μ is the refractive index of the absorbing medium, it is easily seen that the fractional error δ in the path is given by

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* See p. 653.

μ
δ

$$i = \mu\sqrt{(2\delta)}.$$

For instance, if $\mu = 1{\cdot}3$ and $\delta = 0{\cdot}0015$ then $i = 4^\circ\ 6'$. Thus the cell should be set with its plane perpendicular to the light ray to within 4°.

With very little trouble, the cell can be set to 1°, so that the error due to this setting should always be negligible although special attention to this point is required with short cells. The length of cell used should be chosen to obtain a density approximately to the optimum value for the method used; and different cells may often be used with advantage for different portions of the same absorption band.

Effect of temperature. In some cases, absorption varies very considerably with temperature*. It is thus important in any particular case to ascertain by trial if such an effect is present, in which case steps should be taken to ensure that measurements are taken at a temperature sufficiently constant for any error introduced in this way to be negligible compared with other sources of error.

§ 3. THE VARIOUS MEANS OF PHOTOMETRY

We must now consider broadly the means of photometry which are available, and what is the best way to employ each in order to determine extinction coefficients. The means we shall consider are the following: (*a*) Photographed pairs of spectra examined visually; (*b*) photographed pairs of spectra examined by microphotometer; (*c*) visual photometry; (*d*) direct measurement by objective photometer employing photo-electric, selenium or photronic cells, thermopiles or bolometers.

Photographic means of photometry, (*a*) *and* (*b*) *in the above classification.* In photographic methods of spectrophotometry, in which two juxtaposed photographs of spectra are compared visually, the differences in density of blackening which it is possible to detect are of the order of, say, 0·06.

It has long been known that for neighbouring areas of a photographic plate and for a constant time of exposure, the density of blackening of the plate is related to the log of the intensity, I, of the incident light in the way shown by the curves of figure 3.

I

The ratio of the increase of the density of blackening to the increase of light intensity (i.e. the slope of the curve) is greatest on the straight portion of the curve, where it is given a special name, γ. Thus, in making photometric measurements, a plate and method of development (which latter is also very important) giving a large γ is desirable†, as at a, figure 3. On the straight portion of the curve the smallest

γ

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* Mukerji, Bhattacharji and Dahr (*J. Phys. Chem.* **32**, 1834 (1928); *J. Phys. Chem.* **35**, 653 (1931)). In the former paper is recorded a case where a 30° rise of temperature caused an increase of density of about two and a half times.

† The table in the Appendix shows that the most suitable Ilford plate would be the panchromatic half-tone. The Eastman plates recommended are (*a*) Panchromatic-Wratten M (III-B), or alternatively III-F. (*b*) Where panchromatism is not desired, Wratten Metallographic or III-H. (*c*) For the ultra-violet region, III-F, III-H, or III-O, treated by the ultra-violet-sensitizing method. The most suitable developer for all the above plates is that listed by Eastman as D. 19. For brevity we shall subsequently refer to the combination of plate and development simply as "plate."

detectable change in $\log_{10} I$ is obviously given by the smallest detectable difference in density of blackening divided by γ. If a plate having a value of γ equal to 6 be used, then it should be possible to detect a difference in the density of the absorbing substance equal to 0·06 ÷ 6, or 0·01, *and we find that this is confirmed by experiment.*

With a large γ the range of correct exposure (given by the straight portion *cd*, figure 3) is smaller, so that if a plate of large γ be used one must adjust the exposure of the part of the spectrum where a match is being made in accordance with the spectral distribution of the light-source and the sensitivity of the emulsion to different wave-lengths, in such a way that one is working on the portion *cd*, figure 3.

For preliminary surveys one will require a plate with a large latitude *c′d′*, figure 3; that is, a plate with small γ[*]. In this case, however, the measurement of densities will be much less accurate. A list of γ values for various plates and developers is given in the Appendix.

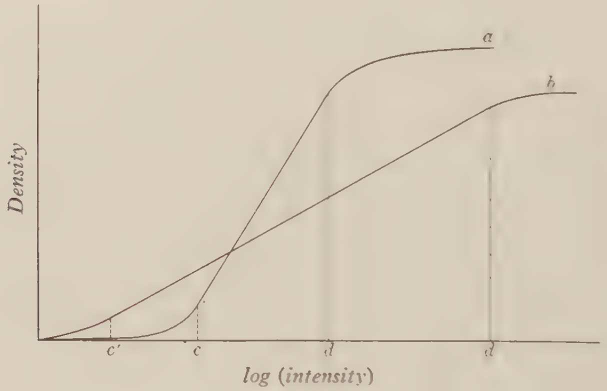

Figure 3.

It has been held that by visual examination of such photographs as those in question, densities can be measured to within 0·02 to 0·04, *independently of the absolute value of density*[†]. As we have seen, this considerably underestimates the accuracy that can be attained, but in any case it remains true that in order that the percentage error in the measurement of the density should be small in photographic spectrophotometry, *the density itself should be large.* It has in the past been very usual to work with densities not exceeding 1·5, but this seems frequently to have been due to the instruments available rather than to any more fundamental reason.

In the use of large densities with a rotating-sector photometer the large number of long exposures required will cause a serious increase in the time required to obtain an absorption curve. Further, the accuracy in making such sectors does not usually justify the measurement therewith of densities greater than 1·5. In using the notched échelon cell*, however, only one exposure is required to obtain a number of readings so that the measurement of relatively large densities does not take unduly long. Further, any error in the rotating sector does not alter the shape for the resulting curve of extinction coefficients, all the ordinates being in error in the same proportion.

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[*] The most suitable Ilford plate here is the Hypersensitive Panchromatic. Eastman Kodak recommend (*a*) Panchromatic—I-F. (*b*) Non-panchromatic— Eastman D.C. Ortho (I-H) or Wratten Hypersensitive. (*c*) Ultra-violet—I-F. A suitable developer in this case is D-76-C.

[†] See H. von Halban and J. Eisenbrand, *Proc. R. S.* A, **116**, 154 (1927).

To take an example, if in the use of an échelon cell of greatest length 1 cm. the density of liquid measured is 0·3, giving us extinction coefficients from 0·3 to 3 and requiring an exposure of about 5 seconds, we may get an error of from 7 to 13 per cent in the measurement of the density, and a like error in the values of the extinction coefficients†. If, however, the density of liquid measured is 1·5, giving extinction coefficients from 1·5 to 15 and requiring an exposure of about $1\frac{1}{4}$ minutes, the error will not be more than from $1\frac{1}{2}$ to 3 per cent. Thus the échelon cell is at its best in conjunction with high extinction coefficients. Where a low extinction coefficient must be measured a photometer which permits the use of a cell of greater length should be used. The employment of photo-electric instead of visual means of comparing the pairs of spectra does not alter the above conclusions.

Dobson‡ describes a method of photometry to which the same fundamental principles apply. In this a neutral-tint wedge is placed in front of the plate or the slit of the spectrograph, the gradient of the wedge being parallel to the length of the slit. A standard source is exposed on each plate as well as the spectra under investigation, and the points at which the density of blackening of the spectrum is equal to that of the standard are determined by means of a photometer. An objection to this method is that points to be compared are not closely adjacent on a plate, so that possible irregularities of development and plate are in this method a source of error.

Visual means of photometry, (*c*) *in the classification on p.* 649. The percentage error in a density-measurement made by a match with the eye is smaller the higher the density§, as in the photographic method, with the following limitations:

(1) The sensitivity of the eye to a given small percentage difference in brightness falls off at low brightnesses. This is specially noticeable at the violet end of the spectrum where the eye, the light-source, and the increased dispersion of the spectrometer act together in producing a small brightness. This last can be overcome without unduly sacrificing the purity of the spectrum, by using for the violet a wider front slit than in other parts of the spectrum. Very great advantage is derived from placing an appropriate filter immediately in front of the slit, or outside the

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* For account of the cell see F. Twyman, L. J. Spencer, A. Harvey, *Trans. Opt. Soc.* **33**, 37 (1931–32); and F. Twyman, *Proc. Phys. Soc.* **45**, 1 (1933).

† Dobson, Griffiths and Harrison, *Photographic Photometry* (1926).

‡ It is assumed that the cell is made or measured with sufficient accuracy for the errors in thickness to be negligible. See p. 648.

§ It easily follows from the Weber-Fechner law that the smallest detectable change of density (ΔD) has a constant value independent of the absolute value of density.

eyepiece in order to absorb the scattered light of wave-lengths which are transmitted freely by the substance under test.

(2) In the case of some visual spectrophotometers it is found possible to obtain an accuracy of about 0·005 in density-measurements by taking a number of observations, this residual error being due to the limit of sensitivity of the eye. Under these conditions, for densities of about 2·0 the scale usually becomes so cramped with forms employing Nicol prisms that there is in addition an error in reading the scale of about the same magnitude (0·005). For this reason densities of between about 1·5 and 2·0 give the best accuracy*.

For the most accurate work a density-reading is accompanied by a reading of the zero for the particular wave-length in question. The accuracy of density-measurement can then be increased by taking readings with the absorbing medium first in one beam and then in the other; the density is half the difference between the two readings. By reading on the opposite side of the zero instead of on the zero itself one obtains twice the accuracy with the same number of readings, i.e. with the same expenditure of time.

$\Delta\theta$ θ ΔI k I

Objective means of photometry, (*d*) *in the classification on p.* 649. One can use a photo-electric method† to measure the absorption of a medium. The sensitivity of such an arrangement is defined by the smallest change in light-intensity which can be measured. In a given arrangement suppose $\Delta\theta$ to be the smallest detectable change in the value of the deflection θ of the galvanometer or other indicating instrument. Then ΔI, the smallest detectable change in light-intensity, is given by $\Delta I = k\Delta\theta$, k being a constant for the particular arrangement. Thus, in photo-electric measurements with a given apparatus ΔI has a constant value independent of the intensity I of the transmitted light, in contrast to the eye and photographic plate, for which, within limits, $\Delta I/I$ has a constant value.

D, ΔD

The condition for greatest accuracy in absorption-measurements is that the percentage accuracy in the measurement of an extinction coefficient shall be a maximum. This implies a minimum value for $\Delta D/D$, where D and ΔD are, respectively, density and the smallest detectable change in density.

I_0, I

If I_0 is the intensity of light incident on the absorbing medium and I the amount transmitted into the measuring apparatus, then the density is given by

$$D = \log_{10} I_0/I \qquad \text{......(1).}$$

On differentiation this becomes

$$\Delta D = -\log_{10} e \,.\, \Delta I/I \qquad \text{......(2).}$$

$$\therefore \quad \frac{\Delta D}{D} = -\frac{\log_{10} e}{ID} \,.\, \Delta I$$

$$= -\frac{\log_{10} e}{I_0 D \,.\, 10^{-D}} \,.\, \Delta I \qquad \text{......(3).}$$

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* The scale of the spectrophotometer described by Dowell in *J. Sci. Instr.* **8**, 382 (1931) and *J. Sci. Instr.* **10**, 153 (1933), which follows a $\tan^4$ law, enables densities above 2 to be used with advantage.

† In which we must be understood also to include the other objective methods mentioned.

To find the optimum value of D we must find when expression (3) has a minimum value, i.e. when $D.10^{-D}$ from (3) has a maximum value.

Thus, differentiating and equating to zero we get $D = 1/\log_e 10 = 0{\cdot}4343$, so that the best density to use is 0·4343. We can see how critical this value is by examining the following values of the expression $D.10^{-D}$ from equation (3).

$D.$	1	0·8	0·43	0·2	0·1
$D.10^{-D}$	0·100	0·127	0·159	0·126	0·079

Thus we may use densities between 0·2 and 0·8 without any considerable loss of accuracy. The above remarks apply in general to photo-electric methods, whether null or deflection methods. But in any particular instrument other features of design might cause one to depart from this condition.

There is no doubt that photo-electric methods give much greater sensitivity of discriminating between two radiations than the eye. The balance of evidence published by those competent to judge seems still on the whole to indicate that as far as absolute measurements are concerned, visual methods are more to be relied upon than photo-electric ones. It should be mentioned too, that besides the electrical circuit, the photometric devices (wedges, sectors, etc.) may also impose limits on both accuracy and sensitivity. The present writers consider, however, that in due course the systematic errors in photo-electric instruments that lead to such a state of affairs will be eliminated.

§4. APPLICATION OF THE ABOVE PRINCIPLES, AND COMPARISON OF THE VARIOUS INSTRUMENTAL METHODS TO SPECIFIC EXAMPLES

The measurement of extinction coefficients in the ultra-violet. We shall assume throughout that we can increase the density of the substance under test to any degree that may be desirable, which can usually be done without difficulty by taking a more concentrated solution or a greater thickness of the absorbing substance.

(*a*) Photographed spectra examined visually. With a quartz spectrograph of the Hilger E. 3 type and the usual sparking set* and photographic plate and development†, and tungsten steel electrodes with 4 mm. spark gap, an exposure of 0·5 sec. suffices to give workable blackening of the photographic plate at wave-lengths from 4600 to 2100 Å. The accuracy of matching of spectra thus produced has been found by us to correspond to a density-difference of 0·01 in the absorbing substance‡. From this we find that for a measured density represented by 1, with exposure-time 5 sec., the limit of accuracy $\Delta D/D$ is 0·01. For double the density the figures are 50 sec. and 0·005 respectively.

(*b*) If the match points of a plate are determined by microphotometer instead of by eye the sensitivity is increased a number of times. At any point, the eye will in general perceive a match over a small range of wave-lengths. If the gradient of the absorption curve at the point is not too great there will be a number of lines in this range, and the microphotometer will find a match on only one (say) of these. In this way it has been found possible* to increase the accuracy of matching by *thirty times*†. In any case where there are insufficient lines for the microphotometer to match on any one line, it may be possible by direct interpolation to determine a point of match between two adjacent lines.

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* As described by F. Twyman and C. S. Hitchen, *Proc. R. S.* A, **133**, 74 (1931), but without inductance in the spark circuit.

† Ilford panchromatic half-tone with developer No. 2 on the list on p. 659.

‡ With such photometric apparatus as that described by F. Twyman, *Trans. Opt. Soc.* **33**, 9 (1931–32).

(*c*) Direct measurement by objective photometer, using the optimum density (see p. 653). The sensitivity is limited by the amount of light which gets through the spectrometer. It may be necessary to open the slits to obtain sufficient light (see p. 647). It has been stated‡ that under favourable conditions the sensitivity may be 10^2 times that of the photographic method in the determination of an extinction coefficient. The method is very much slower than the photographic, and must therefore be looked upon as supplementary to this latter.

There is probably a greater variation of sensitivity between different types of photo-electric instruments than of photographic instruments, and the most important limiting factor in the former is the method of measuring currents. There is probably little to choose in this respect between electrometer methods and valve amplification. With the recently developed "electrometer valves" the valve methods can probably be raised to a further degree of sensitivity than the electrometer methods, but up to the limits of their sensitivity electrometer methods are easier to use.

Measurements in the visible region. In the visible region visual, photographic and photo-electric methods are available, and the choice of a method depends on what accuracy is required, whether time is limited, and the nature of the absorption curve to be measured.

The visual method is the one most usually employed. With polarization spectrophotometers it is possible to make accurate readings up to densities of nearly 2·0 with forms obeying a $\tan^2$ law, or somewhat higher with a $\tan^4$ law.

The photographic method is often quicker. Moreover the whole of the spectrum may be studied on one plate. Reference to p. 653 will show that with the recommended Ilford half-tone plate the accuracy of measurement with the Spekker photometer is about 0·01 in dD/D.

As regards the time required: for measurements at a number of wave-lengths to obtain a complete curve, the photographic method is quicker than a photo-electric measurement when readings have to be obtained and plotted wave-length by wave-length. The photo-electric cell, however, lends itself to automatic reading of spectrophotometric curves. So far as we know this has only been performed in the visible region§, but there is no reason why it should be so limited. This is the next most rapid method of spectrophotometry available, as about half a minute is sufficient to produce a complete curve in the visible region. The sensitivity is found to be limited by the finite thickness of the recording line, among other factors. It is a type of instrument which is extremely useful in industry for comparison, but presents great difficulties if absolute accuracy is sought.

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* The microphotometer employed will be described shortly in another paper.

† The figures can be regarded as representing an increase of sensitivity but not necessarily of absolute accuracy, which latter will depend on the uniformity of cells, sector plates, etc.

‡ H. von Halban and J. Eisenbrand, *Proc. R. S.* A, **116**, 162 (1927).

§ A. C. Hardy, *J. Opt. Soc. Am.* **18**, 97, 165 (1929).

With a visual instrument a number of readings must be taken at each wave-length in order to obtain a mean result of high accuracy. This makes the method more lengthy, but the accuracy is greater than is often obtained by the photographic method.

Measurements in the infra-red. Photographic and photo-electric methods can be applied to the near infra-red up to about 1μ, but in general the thermopile or radiometer are used. The thermopile is most used nowadays. The Müller thermopile is a recent development with a very much greater sensitivity than the established types. It has been found by the present writers to be about 7 times as sensitive as the most sensitive other thermopile with which they are acquainted, and being enclosed it does not suffer much from unsteadiness.

§ 5. THE CHARACTERIZATION OF A RADIATION, SPECTROPHOTOMETRY IN CONNEXION WITH COLOUR, AND THE COMPARISON OF LIGHT-SOURCES

The problems relating to the spectrophotometric comparison of light-sources, including of course sources provided with filters, are in some respects similar to the preceding; but the ensuing paragraph will serve to emphasize points that require special consideration in this type of measurement.

We are now interested in the ratio of the intensities of two radiations, instead of, as in the previous sections, the logarithm of this ratio; and it is now $\Delta I/I$ that defines the sensitivity, where ΔI is the smallest detectable change in the light-intensity I. It should be repeated that within limits, $\Delta I/I$ for the eye and photographic plate is independent of I.

ΔI
I

It is most often the case that the radiations to be measured are continuous spectra in which the rate of change of intensity with wave-length is not so great as it may be in absorption measurements in connexion with chemistry. Thus the considerations which in absorption work call for large dispersion (see p. 647) do not often arise acutely in the work now in question*.

Fluctuations in brightness of a single light-source will not affect the spectrophotometric curve appreciably if they are small; if, however, a comparison between two different light-sources is in question, a curve being plotted wave-length by wave-length as with a visual or photo-electric instrument, any change in intensity of one source during the measurement will result in a curve whose shape is misleading.

Thus it is desirable in such measurements to have the source considerably more steady than it need be in actual use. In the case of electric lamps this result must be achieved by using a very steady electric source such as a battery, or, with similar sources it is sufficient to connect both to the same electric supply.

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* There is a kind of comparison of light-sources which lies outside the intended scope of this paper, namely, the comparison of line spectra, which forms the basis of spectrum analysis. In this of course the dispersion must suffice to separate the lines of the metals which are sought.

Photographed pairs of spectra examined visually or by microphotometer. As has already been seen, the probable error $\Delta I/I$ is independent of the exposure provided the plate be neither under nor over-exposed. To secure highest accuracy the most suitable type of plate is, as before, one with a large γ; with the plate recommended on p. 649 the limit of accuracy $\Delta I/I$ is equal to the limit of discernible density difference 0·01 divided by $\log_{10} e$. That is, an intensity can be determined with this plate with an error of 0·01/0·43 or 2·3 per cent.

It may be pointed out that in quantitative spectrum analysis by emission spectra, for a given method of exciting the spectrum of a given substance, the intensity of a line due to a metallic element is often approximately proportional to the percentage of the element present*, provided that the line is not subject to reversal. One might therefore expect to obtain an accuracy of about 2·3 per cent in the estimation of such an element. So far such accuracy has not been obtained. This may partly be due to limiting accuracy not having been sought, but we attribute it chiefly to the difficulty of running an arc or spark on two separate occasions in such a manner as to give radiations of identical character. Such a constancy would be strange; it is not likely that the various metallic vapours in the arc or spark gap will be present in the same proportions on the different occasions, nor that the electrical conditions of excitation within the gap will remain constant. The selection of Gerlach's homologous line pairs† is a step towards avoiding the latter source of error.

No doubt it was with applications to colorimetry in mind that Guild so strongly disfavoured that class of instrument (which we designate the "first type") in which "the observer views through the eyepiece of a spectroscope two adjacent spectra, one above the other, isolating a small wave-length range by means of adjustable shutters in the eyepiece‡," and favours a second type in which "the observer looks through a slit situated in the plane of the spectra...and sees a suitably divided field"‡ of the dimensions generally accepted for photometric work§. The essential difference between the two types is that in the first the available light is concentrated into a slit image which constitutes the photometric field, while in the second type the available light is distributed over a relatively large photometric field.

It must be remembered that for large densities and with slits narrow enough to give a sufficiently pure spectrum the amount of light available is very small, and the field of view will in no case be very bright. In each type mentioned there are two opposing features which respectively increase and decrease the accuracy of reading attainable. Thus the greater brightness of the field in the first type, as compared with that in the second, enhances the accuracy of readings. On the other hand, for a constant brightness the standard field of the second type allows of more accurate reading than the narrow-slit field of the first type.

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* F. Twyman and A. Harvey, *J. Iron and Steel Inst.* No. 11, 397 (1932); F. Twyman and C. S. Hitchen, *Proc. R. S.* A, **133**, 87 (1931).

† F. Twyman and A. Harvey, *loc. cit.* p. 399.

‡ J. Guild, *Proc. Opt. Convention* 1926, pp. 21, 22.

§ A field of 2° has been recommended by Ives for flicker photometry and by Guild for colorimetry, *ibid*, pp. 36, 72.

It is not obvious at first sight which of the two factors has the greatest effect and it is a matter of experiment to determine which of the two types will be better.

A Hilger-Nutting spectrophotometer (an instrument of the first type) is easily converted into one of the second type by covering one-half of the object-glass of the spectrometer telescope with a prism of small angle, about 9′, so that spectra normally closely adjacent become accurately superposed. On then removing the eyepiece and viewing the object-glass through the slit one sees the type of field used in the second class. In this way a very direct comparison of the two types was made. With various widths of the collimator slit and a telescope slit-width of 5 Å. at the D lines readings were made on a density of about 1·7. At a wave-length of 5890 Å. it was found possible for a skilled observer with great concentration* to take readings having a mean error of 0·03 in density with a collimator-slit opening of 4 Å. By means of the slit field, however, it was possible to get readings with a mean error of less than half of this with a slightly smaller width of collimator slit (3 Å. at the D lines). Moreover it was possible to take these readings with far greater comfort, an important factor in taking a large number of readings.

Apart from this, in the second type there will be a tendency to obtain sufficient brightness by opening the telescope slit; and the use of an eyepiece to observe and limit this width according to the gradient of absorption is an additional adjustment and one liable to be overlooked in routine work by some of those who have to use spectrophotometers. Thus, as the result of experiment, it would seem that for accurate measurements of high density spectrophotometers using a slit field are not merely as good as the other type, but definitely preferable to it.

Photo-electric measurements. It appears that for measurements of this character photo-electric methods are peculiarly appropriate, for in these I is a constant so that by making I very large we can make the error in measuring $\Delta I/I$ very small.

Choice of a method for a particular purpose in this section. In comparison of light-sources which are liable to be unsteady a photographic method is preferable, so that exposure for the whole spectral range is made at the same time. We can then derive from the measurements a curve of comparison of the light-sources which is true at the time of taking the photograph. If comparison is made point by point, any individual measurements may be subject to a variation from the average value peculiar to the moment at which the reading was taken. If, however, a more sensitive photo-electric arrangement is available, it will be of advantage to use it provided that the variation of the source can be kept less than is detectable with the photographic plate.

The choice of a suitable method for other purposes in this section will depend on considerations similar to those already elaborated at the end of § 2, to which reference should be made.

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* The difficulties of observing this field are threefold: (1) the very small brightness; (2) the very narrow eye-pupil necessitates keeping the eye very steady; (3) some diffusion of the image arises owing to the very narrow eye-pupil.

§ 6. ACKNOWLEDGMENTS

The thanks of the authors are due to Dr Simeon, Mr Perry, Mr Follett, Mr Spencer and Dr Harvey of the staff of Adam Hilger, Ltd., for help in connexion with various points which arose in the writing of this paper.

APPENDIX

Table 1. γ values for various plates, obtained by using the developers 1, 2, 3 described below at 65° F.

Time of development (minutes at 65° F.)	γ values obtained for light of the quality specified			
	White light; colour temperature 2360° K. (vacuum lamp)	White light (2360° K.) screened by filters		
		Tricolour blue 3900–5150 Å.	Tricolour green 4800–6100 Å.	Tricolour red 5800 Å.–red end
	Ilford Panchromatic Half-tone Developer no. 2: caustic potash, hydroquinone			
2	6–7	5–6	6–7	8·5–9·5
3	7–8	6–7	8–9	9–10
	Ilford Rapid Process Panchromatic Developer no. 2: caustic potash, hydroquinone			
2	3–4	2·5–3·5	2–3	3–4
3	4–5	2·5–4	3–4	4–5
	Wellington Soft Spectrum Developer no. 1: metol, hydroquinone			
5	1–1·5	0·6–0·9	1–1·5	1–1·5
7½	1·25–1·75	0·80–1·2	1·25–1·75	1·25–1·75
	Ilford Hypersensitive Panchromatic Developer no. 1: metol, hydroquinone			
5	0·5–0·7	0·4–0·6	0·5–0·7	0·5–0·7
7½	0·6–0·8	0·5–0·7	0·6–0·8	0·6–0·8
	Wellington Anti-Screen Developer no. 1: metol, hydroquinone γ obtained on exposure to light of mean noon sunlight quality (Washington)			
5	1·25–1·75			
	1·50–2·0			
	Ilford Special Lantern plate Developer no. 3 Exposure to light from unscreened vacuum lamp at 2360° K.			
2	1·75–2·0			
3	2·00–2·5			

We have to thank Ilford, Ltd., for very kindly supplying the information contained in the above table.

Table 2. Methods of development

(1) Metol, hydroquinone.

Metol	2 gm.
Sodium sulphite (crystals)	100 gm.
Hydroquinone	8 gm.
Sodium carbonate (crystals)	100 gm.
Potassium bromide (10 per cent solution)	8 cm^3
Water up to	1000 cm^3

For use take 1 part with 2 parts of water.

(2) Caustic potash, hydroquinone.

A.	Caustic potash	50 gm.
	Water up to	1000 cm^3
B.	Hydroquinone	25 gm.
	Potassium metabisulphite	25 gm.
	Potassium bromide	25 gm.
	Water up to	1000 cm^3

For use take equal amounts of A and B.

(3) Developer for black tone (rapid) lantern plates.

A.	Hydroquinone	9 gm.
	Sodium sulphite (crystals)	50 gm.
	Water up to	500 cm^3
B.	Caustic soda	5 gm.
	Potassium bromide	2 gm.
	Water up to	500 cm^3

For use take equal amounts of A and B.

DISCUSSION

Mr J. Guild: The authors have drawn explicit attention to several important considerations which, while familiar enough to those who have had to give special attention to these problems, are apt to be overlooked by those who merely *use* spectrophotometers, and sometimes also, though less excusably, by those who design them.

I have three minor criticisms to offer; two of a general character and one of a specific character. The first is that although the authors are careful to draw attention in several places to the important distinction between precision and accuracy, their specific conclusions regarding the potentialities of different methods are based entirely on criteria of precision. Nowadays the experienced experimenter in all branches of experimental science attaches much less importance to precision than was attached in earlier days, when all apparatus was relatively crude and its precision was the governing factor in determining its utility. Now, however, Messrs Hilger and other similarly progressive instrument makers provide us with such sensitive apparatus of all kinds that the reproducibility of the results we can obtain with them is no longer a useful criterion of the value of these results, which is generally determined by other considerations altogether. These considerations are not so readily reducible to formulae, and so are usually ignored in any formal treatment of the merits of equipment or methods; but to ignore them is dangerous. No experienced worker ignores them in considering the equipment or methods which he will use himself, and so he is guilty of suppressing something which he knows to be important if he ignores them in instructing others.

If two instruments are to be used under practically identical conditions, all the essential operations involved in their use being the same in each case, then the more precise instrument is the better one for the job in question; but if the use of one instead of the other involves essential alterations in the conditions of experiment, so that factors have to be taken into account in one case which do not have to be considered in the other, then a judgment based on relative precision may be entirely misleading.

This is well illustrated by the present state of affairs in this very subject of spectrophotometry. The visual method is among the least precise of available methods, yet, curiously enough, at the present time it is the method on which most reliance is placed by the spectrophotometrists of the various National Laboratories, despite the fact that alternative methods of much greater precision are available in most of these institutions. This predilection is based on miscellaneous experience, and in its justification can be adduced the indisputable fact that there are far more spectrophotometric data, of verified high accuracy, for the range of wave-lengths within which the visual methods are applicable than for the ultra-violet and infra-red regions where other methods must perforce be employed.

This is not necessarily a permanent state of affairs. It is the aim of most spectrophotometrists to bring physical methods to a degree of reliability which will enable them to supersede visual methods altogether, but this has not yet been done, despite the extravagant claims, based on sensitivity, put forward by various inventors of physical spectrophotometers; and the present state of affairs, which we all hope will become a past state in the near future, shows of how little use precision is as a guide to the best available method. The photographic method, which ranks fairly high if we take as criterion the sensitivity figures given in this paper, ranks very low in the estimation of spectrophotometrists. In the National Physical Laboratory we never use it except as a last resort in cases where we cannot apply any other method, as, for instance, when light-sources of an unsteady character are involved. While I cannot speak with certainty for others on this point, I should be very greatly surprised if this were not also the status of the method in other standardizing institutions.

The second general criticism which I have to offer is that by drawing a distinction between the problems of measuring density and measuring transmission the authors may lead an unwary reader into the pitfall of confusing arithmetic with physics. The process of spectrophotometry is always the measurement of relative quantities of light, not darkness. In the case of a slab of material the quantity measured is transmission, never anything else. The problems of measurement are not affected when, for some purpose, it becomes convenient to invert the answer and take its logarithm. The method, whatever it may be, which for a given specimen will give the greatest accuracy in the transmission at a given wave-length is clearly giving the most accurate information about the specimen, whatever function of the result we are interested in. I am quite certain the authors themselves will agree with this; but I can easily imagine a reader concluding from the paper that for a given specimen the best method of spectrophotometry to use depends on whether he is interested in the light which the specimen lets through or the light which it stops.

Although it is not quite relevant to this paper, which is not concerned with nomenclature, I should like to deprecate most strongly the habit of physical chemists and others of inventing new derived magnitudes, such as density, in order to convert multiplication into addition. It is quite possible, after all, to use logarithms as a legitimate aid to arithmetic without inventing a new magnitude to be represented by the logarithms of the magnitudes with which we are really concerned. Our terminology should be kept in as close correspondence as possible with the physical magnitudes actually handled in our experiments.

The third point to which I wish to refer is the only one on which I would differ from the authors on a question of fact. In § 5 reference is made to certain views expressed by me on the merits of two types of visual spectrophotometer. The authors find themselves in agreement with these views only as regards cases where comparatively transparent specimens are involved so that there is plenty of light; and, with characteristic courtesy, express the belief that I had only such cases in mind. This is not so: I intended the opinion in question to be as general and sweeping as possible, and if it can be shown to be wrong in any circumstances whatever then I was just wrong. I am still of the opinion, however, that instruments of the "second type," giving a Maxwellian view, are superior to those of the "first type" as exemplified in the Nutting spectrophotometer in all circumstances, despite the evidence which the authors adduce to show that when the available intensity is very low the latter type is better. I do not consider that a fair comparison can be made merely by adapting an instrument designed for use in one way so that it can be used temporarily in another way and then regarding its behaviour as typical of the merits of these two ways. The question is how instruments, properly designed to obtain the best possible results by each method, compare with one another. In so far as I am capable of interpreting performances expressed in terms of density, there is no doubt at all that with the apparatus used at the National Physical Laboratory a precision very much superior to the figures obtained in the authors' tests is obtained with very much denser specimens than those mentioned. As a matter of fact, it was the inadequacy of the Hilger-Nutting spectrophotometer for use when illumination is low—as it always is at the ends of the spectrum, however transparent the specimen under test may be—which led me to make the categorical statement referred to.

A further disadvantage of this type of instrument is that in cases where the transmission curve is sloping very steeply, the mode of subjective integration which is operative within the range of wave-lengths comprised by the telescope slit is unknown, and the necessary slit-width correction cannot be evaluated unambiguously. With the Maxwellian view, on the other hand, the slit-width correction can always be quite definitely computed.

Authors' reply. We have reviewed our argument in the light of Mr Guild's criticisms, and although we are in general agreement with what is said, we still find no escape from our conclusions. Two specific points that are raised appear to call for further comment. The term "density" is employed as a measure of absorption. Absorption is not merely a derived magnitude but a phenomenon of direct physical significance and of considerable importance in some applications of spectrophotometry. We do not think Mr Guild would advocate also the abolition of the term "absorption," and it is clear that, when the emphasis is on this aspect of the measurements, to refer shortly to "density" saves considerable circumlocution.

In regard to the comments on the accuracy attainable, it must not, of course, be forgotten that the Hilger-Nutting spectrophotometer, in common with other instruments, has improved considerably since the date of Mr Guild's first experience with it.

The accuracy obtained by Mr Guild is surprisingly good and seems better than what is generally considered possible, considering the known limitations of the human eye in the discrimination of intensity.

We value the remarks of Mr Guild on this subject and are very pleased to have them associated with our paper for the benefit of those interested in present-day developments in spectrophotometry.

535.88

THE VISIBILITY OF OBJECTS IN A SEARCHLIGHT BEAM

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ABSTRACT. The paper is almost entirely mathematical, and calculates the effect on the visibility of an object, illuminated by a projector, of the scattering and absorption of the light by the atmosphere. The discussion is divided into three parts dealing separately with the light reflected from the object from the incident beam, with the brightness of the atmospheric background due to scattered light, and of the conditions of visibility as functions of the two previous quantities. Numerical solutions of the equations are given and range curves are drawn for objects of different sizes and different reflecting powers and for sources of varying intensity.

§ 1. INTRODUCTION

It is well known that the range of a searchlight is limited by two main factors. The first is the attenuation of the illumination due to scattering and absorption in the atmosphere and to the spread of the beam, which varies inversely as the square of the distance between source and object. The second is the background of light due to the flux scattered by the atmosphere from the beam of the searchlight itself. Attempts have been made to calculate these two effects from fundamental considerations, but the author is unaware of any publication which has carried the investigation to the stage of drawing (intensity, range) curves. This is what is now attempted on the basis of certain definite assumptions.

The subject of discussion is divided into three main parts: (1) the light reflected from the object from the incident beam; (2) the brightness of the atmospheric background due to light scattered from the incident beam; (3) the conditions of visibility as functions of (1) and (2) above.

Each of these sections being taken in turn, the assumptions involved may be stated. In (1) it is assumed that a reflecting surface which subtends an angle so small as to be below the limit of resolution denotes its presence only by the light reflected from it, and that the effect on the eye is the same for such a secondary source as it is for a primary source of the same intensity. This is probably justified. In (2) it is assumed that no light is absorbed by the atmosphere but that all the light that does not reach the spherical surface which includes the object and has the source as centre is scattered. If any light is absorbed, as is certainly the case in practice, the scattered light will be correspondingly reduced and the background brightness diminished. The further simplifying assumption is made that the scattering is uniform in all directions. Rocard* calculates that the forward scattering is some ten times the backward scattering, so that this assumption also tends to make the background brightness higher than it should be with the result that the range, as calculated later, tends to be low. Thus, it is probable that the calculated range figures deduced are minima. In (3), for simplicity, an empirical relation between the minimum illumination needed for conspicuity and the background brightness is taken. It can only be stated that the values given by this equation are in general agreement with data available in this country and are, of course, justified by the results obtained by the authors of the expression.

§ 2. THE LIGHT REFLECTED FROM THE OBJECT

In figure 1 let B be the object in the beam of the projector placed at A and let an observer be placed at C.

R Let R be the range AB of the light;
δ δ the distance AC between projector and observer;
I_0 I_0 the initial luminous intensity in the beam;
I I the effective intensity at distance R; and
κ κ the fraction of light absorbed and scattered per unit distance.

Then

$$I = I_0 e^{-\kappa R} \qquad \text{......(1).}$$

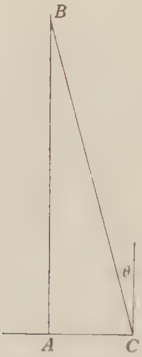

Figure 1.

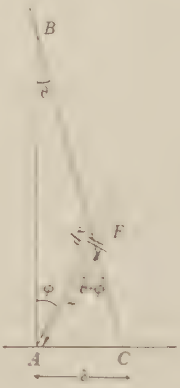

Figure 2.

ρ The illumination on the object at B is I/R^2, and if ρ is the diffuse reflection factor of the object (assumed to be perfectly diffusing but grey) its brightness is given by

$$\rho I_0 e^{-\kappa R}/\pi R^2.$$

Since the object as seen from C has to send its reflected light through a distance
b BC, its apparent brightness b is then given by

$$b = \rho I_0 e^{-\kappa(R+BC)}/\pi R^2 = \rho I_0 e^{-\kappa R(1+\sec\theta)}/\pi R^2$$

$$\fallingdotseq \rho I_0 e^{-2\kappa R}/\pi R^2 \quad \text{when } \theta \text{ is small} \qquad \text{......(2)†.}$$

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* Rocard, *Revue d'Optique*, No. 5, p. 202 (1932).

† See A. Blondel, *Ill. Eng.* **8**, 87 (1915).

Now the brightness of an object is only of interest if the object is large, that is to say, in practice, if it subtends an angle greater than, say, 2° at the eye of the observer. If the subtended angle is small, the important property is the total light-intensity in the specified direction. Thus, let a be the area of the object, a
assumed as square for simplicity.

Then the intensity i in the observer's direction is given by i

$$i=\frac{a\rho I_0 \cos\theta \,.\, e^{-\kappa R}}{\pi R^2},$$

and the illumination e_1 produced at the observer's eye is given by e_1

$$e_1=ie^{-\kappa(BC)}/(BC)^2;$$

or since $BC=R\sec\theta$,

$$e_1=\frac{a\rho \cos^3\theta \,.\, e^{-\kappa R(1+\sec\theta)}}{\pi R^4},$$

or when θ is small,

$$e_1=\frac{a\rho I_0 e^{-2\kappa R}}{\pi R^4} \qquad \text{......(3).}$$

If the object subtends a constant angle ψ at the observer's eye ψ

$$e_1=\frac{\psi\rho I_0 \cos\theta \,.\, e^{-\kappa R(1+\sec\theta)}}{\pi R^2},$$

or

$$e_1 \doteqdot \frac{\psi\rho I_0 e^{-2\kappa R}}{\pi R^2} \quad \text{when } \theta \text{ is small.} \qquad \text{......(4).}$$

§ 3. THE BRIGHTNESS OF THE ATMOSPHERIC BACKGROUND DUE TO SCATTERED LIGHT

In figure 2, consider an elementary volume of air at F, of unit area and thickness dx, distant x from A. The light falling on this unit is $I_\phi e^{-\kappa x}$ and the light scattered x
in the thickness dx is

$$\kappa I_\phi e^{-\kappa x} x^{-2}\,dx.$$

Since this is assumed to be scattered uniformly, the brightness in any direction is $(\kappa/4\pi)\,I_\phi e^{-\kappa x}x^{-2}\,dx$. The thickness of the layer as seen from C is $dx\sec(\theta+\phi)$, and the apparent brightness is

$$(\kappa/4\pi)\,I_\phi e^{-\kappa(x+FC)}\,x^{-2}\sec(\theta+\phi)\,dx \qquad \text{......(5).}$$

From the figure,

$$R\sin\theta=x\sin(\theta+\phi)$$

and

$$R\tan\theta=\delta,$$

whence

$$\delta=x\sin(\theta+\phi)\sec\theta.$$

Also

$$FC=BC-BF=\frac{\delta}{\sin\theta}-\frac{R\sin\phi}{\sin(\theta+\phi)},$$

whence, on reduction, $x+FC=\delta(\cos\theta+\cos\phi)\operatorname{cosec}(\theta+\phi)$.

Also $x=\delta\cos\theta\operatorname{cosec}(\theta+\phi)$ and $dx=\delta\cos\theta\cos(\theta+\phi)\operatorname{cosec}^2(\theta+\phi)\,d\phi$.

Substitution in (5) gives

$$(\kappa/4\pi\delta)\, I_\phi e^{-\kappa\delta\,(\cos\theta+\cos\phi)\,\mathrm{cosec}\,(\theta+\phi)} \sec\theta\, d\phi\,;$$

or the total brightness

B

$$B=\frac{\kappa}{4\pi\delta\cos\theta}\int_{-\theta}^{\pi/2} I_\phi e^{-\kappa\delta\,(\cos\theta+\cos\phi)\,\mathrm{cosec}\,(\theta+\phi)}\, d\phi \qquad \text{......(6)*}.$$

ϕ_1

For the case of a concentrated beam, such as that of a searchlight, where the angle of the beam may be taken as $\pm\phi_1$ and I_ϕ may be taken to a first approximation as constant over this angle, and θ is small:

$$B=\frac{\kappa I_0}{4\pi\delta}\int_{\substack{-\phi_1\\ \text{or } -\theta}}^{\phi_1} e^{-2\kappa\delta\,(\theta-\phi)}\, d\phi \qquad \text{......(7)}.$$

§ 4. THE VISIBILITY OF OBJECTS IN THE BEAM

For objects whose angular subtense approximates to 2°, the visibility is governed by the ratio $b\ B$ which becomes in effect the Fechner fraction. This has been shown to vary with B†, but since a 2° object at 5000 ft. would be 170 ft. long or 29,000 ft.² in area, it is clearly not a problem which frequently arises.

If the Fechner fraction were truly constant the range of a searchlight would be independent of its intensity since (2) ÷ (7) is independent of I_0. Thus,

$$\frac{\rho I_0 e^{-2\kappa R}}{\pi R^2}\Big/\frac{\kappa I_0}{4\pi}\int e^{-2\kappa\delta/(\theta+\phi)}\, d\phi=\frac{4\rho\delta e^{-2\kappa R}}{\kappa R^2\int e^{-2\kappa\delta/(\theta+\phi)}\, d\phi}=\Phi \qquad \text{......(8)},$$

where Φ is Fechner's fraction.

This fraction is, however, constant over a considerable range of background brightness, and for this condition R may be computed from (8).

The more important practical case is that of smaller sources than would subtend 2° at the observer's eye. Langmuir and Westendorp‡ have given an empirical formula, which is sufficiently accurate over the range required, indicating the relation between the background brightness and the illumination produced at the observer's eye by the source needed for adequate conspicuity. The formula is

$$e_1 = 1{\cdot}08\times 10^{-7}\sqrt{B} \qquad \text{......(9)},$$

where e_1 and B have the meanings already allocated. e_1 is sufficient for foveal vision when B exceeds about 10^{-3} candles/ft.²

Thus, when θ is small, by combination of (3), (7) and (9)

$$\frac{a\rho I_0 e^{-2\kappa R}}{\pi R^4}=1{\cdot}08\times 10^{-7}\sqrt{\left\{\frac{\kappa I_0}{4\pi\delta}\int_{\substack{-\phi_1\\ \text{or } -\theta}}^{\phi_1} e^{-2\kappa\delta\,(\theta+\phi)}\, d\phi\right\}},$$

whence

$$\frac{e^{-2\kappa R}}{R^4}=\frac{0{\cdot}54\times 10^{-7}}{a\rho}\sqrt{\left\{\frac{\kappa\pi}{\delta I_0}\int_{\substack{-\phi_1\\ \text{or } -\theta}}^{\phi_1} e^{-2\kappa\delta/(\theta+\phi)}\, d\phi\right\}} \qquad \text{......(10)},$$

from which R may be computed.

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* For a similar expression, see Rocard, *loc. cit.* p. 207.

† W. S. Stiles, "The effect of glare on the brightness difference threshold," *Dept. Sci. Ind. Res.* Paper no. 8 (p. 23).

‡ *Physics*, **1**, 273–317 (1931).

§ 5. NUMERICAL SOLUTIONS

For these solutions δ has been taken as 100 ft.;

ϕ_1 as 0·020 radians; and

ρ as 1·0.

Case 1. $\kappa = 0$, i.e. the atmosphere is perfectly clear.

It follows that $B = 0$ and (7) and (10) are therefore useless. It is known, however, that when $B = 0$, $e_1 = 0{\cdot}54 \times 10^{-8}$ foot-candles.

Then from (3)

$$R^4 = aI_0/0{\cdot}54 \times 10^{-8}\pi.$$

Values of R from this for various values of I_0 and a are given in table 1.

Table 1.

I_0 (candles × 10^6)	R (hundreds of feet)			
	$a = 200$	$a = 100$	$a = 50$	$a = 25$
100	329	277	233	196
200	390	328	276	232
400	465	392	330	278
600	514	433	364	307
800	554	466	392	330
1000	583	491	413	347
1500	645	543	457	384

Case 2. $\kappa = 0{\cdot}0000095$, giving 95 per cent transmission per mile as in normal clear weather.

The values of the integral $\frac{1}{\delta}\int_{-\phi_1 \text{ or } -\theta}^{\phi_1} e^{-2\kappa\delta/(\theta+\phi)}\,d\phi$ in equations (7) and (10), which we shall denote by A_1, have been evaluated graphically and are shown in table 2. A_1
In this process $-\phi_1$ or $-\theta$, whichever is the smaller, is taken as the lower limit.

Table 2.

R (hundreds of feet)	θ (radians)	Limits of integral (radians)	Values of integral A_1
10	0·100	0·020 to − 0·020	0·000391
20	0·050	0·020 to − 0·020	0·000383
30	0·033	0·020 to − 0·020	0·000362
40	0·025	0·020 to − 0·020	0·000343
50	0·020	0·020 to − 0·020	0·000327
100	0·010	0·020 to − 0·010	0·000233
150	0·007	0·020 to − 0·007	0·000200
200	0·0050	0·020 to − 0·0050	0·000181
300	0·0033	0·020 to − 0·0033	0·000170
400	0·0025	0·020 to − 0·0025	0·000165
500	0·0020	0·020 to − 0·0020	0·000162
1000	0·0010	0·020 to − 0·0010	0·000155
2000	0·0005	0·020 to − 0·0005	0·000150
∞	nil	0·020 to ∞ nil	0·000141

Then the background brightness in candles ft^2 is given by 0·0000095 $A_1 I_0$ 12·56 for various values of R and I_0. These are shown in table 3.

Table 3.

I_0 (candles × 10^8)	B (candles/ft^2)	
	R = 1000	$R = \infty$
1	0·028	0·010
2	0·059	0·021
4	0·12	0·042
6	0·18	0·063
8	0·23	0·085
10	0·29	0·105
15	0·44	0·160

Thus for this weather condition and when I_0 is between 100 and 1500 million candles, the background brightness only varies from 0·010 to 0·440 candles ft^2. The value of the Fechner fraction varies from 0·8 to 1·2 per cent.

For small sources R is given by the expression

$$\frac{e^{-2\kappa R}}{R^4} = \frac{1{\cdot}08 \times 10^{-7}}{2a}\sqrt{\frac{0{\cdot}0000095\pi A_1}{I_0}} = 2{\cdot}95 \times 10^{-10}\sqrt{\frac{A_1}{I_0 a^2}}$$

where A_1 is given above.

Graphical solution only is possible and figure 3 gives values of the left-hand side for various values of R. From this curve are obtained the solutions shown in table 4.

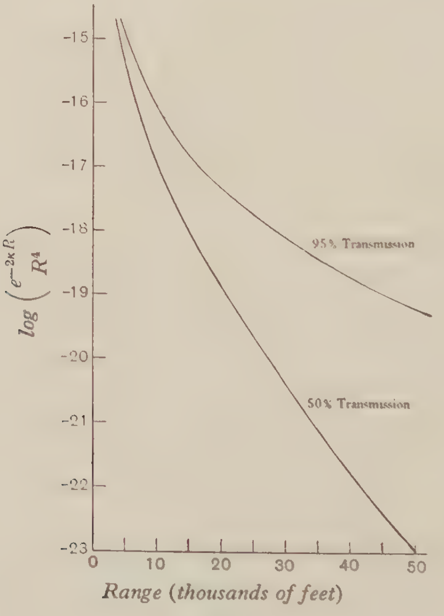

Figure 3.

Table 4.

I_0 (candles × 10^8)	R (hundreds of feet)			
	$a = 200$	$a = 100$	$a = 50$	$a = 25$
1	250	210	180	150
2	270	232	195	165
4	290	250	210	180
8	313	265	230	190
15	336	290	250	210

Case 3. Transmission 50 per cent per mile; $\kappa = 0{\cdot}000095$.

Table 5. Values of integral A_2, viz. $\frac{1}{\delta}\int_{-\phi_1 \text{ or } -\theta}^{\phi_1} e^{-2\kappa\delta/(\theta+\phi)}\,d\phi$. A_2

R (hundreds of feet)	A_2	R (hundreds of feet)	A_2
10	0·000327	200	0·000050
20	0·000276	300	0·000040
30	0·000215	400	0·000039
40	0·000166	500	0·000038
50	0·000131	1000	0·000035
100	0·000075	2000	0·000032
150	0·000055	∞	0·0000309

The values of the background brightness vary from 3·7 candles/ft.² (when $R = 1000$, $I_0 = 15 \times 10^8$) to 0·047 candles/ft.² (when $R = \infty$, $I_0 = 10^8$).

For small sources $e^{-2\kappa R}/R^4 = 2{\cdot}95 \times 10^{-9}\sqrt{(A_2/a^2 I_0)}$.

Table 6.

I_0 (candles × 10^8)	R (hundreds of feet)			
	$a = 200$	$a = 100$	$a = 50$	$a = 25$
1	100	90	80	70
2	110	100	85	75
4	120	105	90	80
8	130	110	95	85
15	135	115	100	90

Any change in a and ρ give corresponding effects on the range, i.e. halving the area has the same effect as halving the reflecting power. This is illustrated in table 7, where $I_0 = 2 \times 10^8$ and the atmospheric transmission is 95 per cent.

Table 7.

ρ	R (hundreds of feet)			
	$a = 200$	$a = 100$	$a = 50$	$a = 25$
1·0	270	232	195	165
0·5	232	195	165	137
0·25	195	165	137	117
0·125	165	137	127	100

Table 8. $I_0 = 200 \times 10^6$; the atmospheric transmission is 50 per cent.

ρ	R (hundreds of feet)			
	$a = 200$	$a = 100$	$a = 50$	$a = 25$
1·0	110	100	90	80
0·5	100	90	80	67
0·25	90	80	67	60
0·125	80	67	60	52

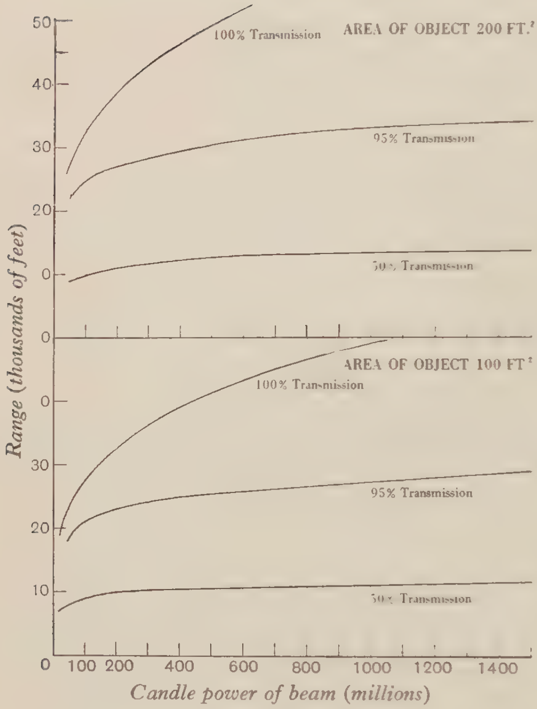

Figure 4 (*a*).

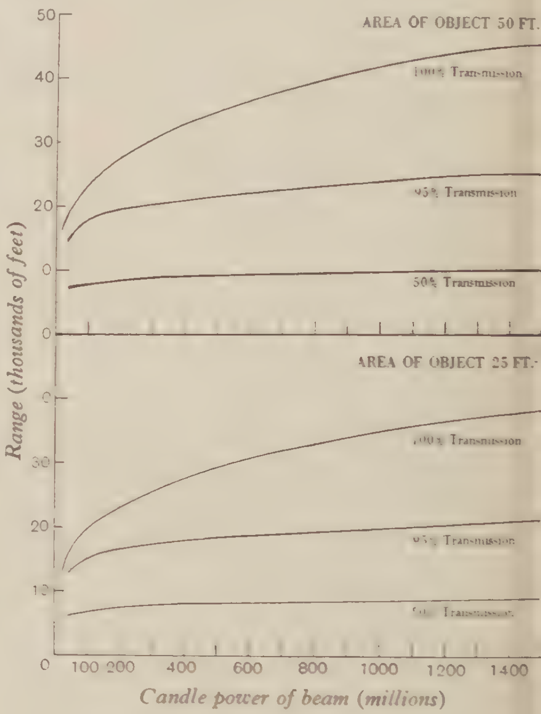

Figure 4 (*b*).

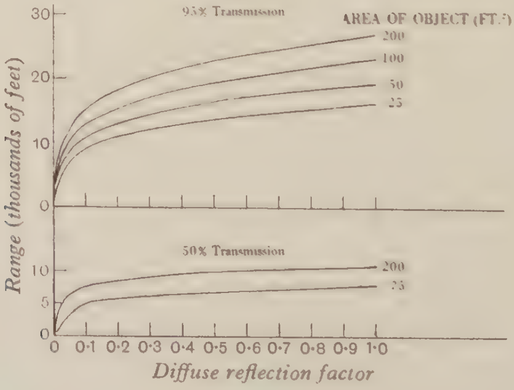

Figure 5.

§ 6. CONCLUSIONS

Increase in intensity has a decreasing effect on the range of the searchlight as the intensity increases. The range becomes more nearly independent of the intensity as the transmission of the atmosphere decreases. The range increases as the area of the object increases, though not proportionally, provided the object is sufficiently small to be considered as a point source.

DISCUSSION

Mr H. G. **Bennett**. The author's results agree fairly well as regards order of magnitude with some experimental results given by Messrs Sperry in a recent catalogue of optical equipment. The most striking difference between the two sets of results is at the lower beam-intensities. Messrs Sperry find that increases of intensity have a marked effect on range up to, say, 500 million c.p., whereas the author finds that the effect becomes much reduced above, say, 200 or 300 million c.p. Such differences are to be expected when it is realized that the conditions of experiment probably did not correspond with the hypotheses of the theory in a number of respects. This being so, it is remarkable that such near agreement at the higher intensities was obtained.

These considerations lead me to ask the author how much his results would have been affected had he taken other values for the threshold, the distribution of light in the beam, etc. Workers in visibility know only too well how small differences in the conditions may affect one's ability to see, and if the author would answer this question it would greatly help in the comparison of his theory with practical results.

Mr J. H. **Awbery**. I would refer to § 5 on "numerical solutions." Immediately below table 1 is the quantity A_1 or $\frac{1}{\delta}\int_{\alpha}^{\phi_1} e^{-2\kappa\delta/(\theta+\phi)}\,d\phi$, which is stated to have been evaluated graphically. In general, I am convinced that numerical methods, such as Simpson's rule, give not only more accurate but also more rapid solutions to such problems. In this particular case, A_1 can be reduced to tabulated functions by taking $2\kappa\delta/(\theta+\phi)$ as a new variable. Then $A_1/2\kappa = [e^{-y}/y + Ei\,(-y)]$, taken between the limits $y = 2\kappa\delta/(\theta+\phi_1)$ and $2\kappa\delta/(\theta+\alpha)$. The *Ei* functions are tabulated in Jahnke and Emde's *Funktionentafeln*, p. 21*.

On the next page of the paper the equation $e^{-2\kappa R}/R^4 = CA_1^{\frac{1}{2}}/aI_0^{\frac{1}{2}}$ has to be solved for a number of values of a and I_0. When κ and δ are kept constant each side of the equation is a function of R only, since A_1 depends on that variable. From figure 3 it appears that the equation was solved by finding the intersections of the two curves representing them. This involves drawing a fresh curve to represent the right-hand side for each pair of values of a and I_0. A much less tedious process would have been to plot $e^{-2\kappa R}/R^4 A_1^{\frac{1}{2}}$, in which case only one curve would be needed.

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* B. G. Teubner (Berlin, 1923).

The value of R would simply be read off opposite the value of $C\ aI_0^{\frac{1}{2}}$. Incidentally, this equation shows that R is unchanged by any alteration of a and I_0 which leaves $I_0 a^2$ unaffected. This is only approximately so in table 4, as may be seen by comparing the entries in the line for $I_0 - 2$ with those in the next succeeding columns, for $I_0 = 8$. It may be worth remarking that if the integrations are more accurately carried out the numbers in table 4 are altered by less than 5 per cent (downwards).

AUTHOR'S reply. I am obliged to Mr Awbery for his interest in my paper, especially as he was moved to recalculate so many of the figures. I was unaware of any tabulated functions which would assist in the necessary calculations, although I searched the tables of integrated functions which were known to me. I agree that the methods suggested by Mr Awbery yield useful results as he indicates, but it appeared simpler to me to make use of graphical integration.

In reply to Mr BENNETT: It was the general agreement of my calculated results with those published by Messrs Sperry that encouraged me to proceed with the presentation of this paper. I have not investigated the effect of variations in the threshold values of illumination on the ranges given, because the experimental (illumination, background-brightness) curve for the observer would have to be used to find how such changes affected the results. The same remark applies to the distribution of light in the beam. Given the polar curve of a searchlight, graphical or numerical integration would give the background-brightness from the equations provided, and I suggest that it is for others who have experimental range results to check up particular cases in order to confirm or reject the relatively simple theory I have put forward.

551.594.11 551.51.053.5

ON TWO METHODS OF IONOSPHERIC INVESTIGATION

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ABSTRACT. Two wireless methods of measuring upper-atmospheric ionization, both of which involve measurements of the equivalent height of reflection for a number of electric wave frequencies, are described. Different relations are expected and found to exist between the equivalent height and the frequency for the ordinary and extraordinary magneto-ionic components. From the examples chosen to illustrate the methods the following deductions and measurements concerning ionospheric characteristics are made. (*a*) Magneto-ionic double refraction is caused by region E; this shows, as was previously demonstrated in the case of region F, that free electrons are the effective electrical agents. (*b*) For daytime conditions, and more frequently in summer than in winter, evidence of the existence of a protuberance or "ledge" on region F is found, which most probably indicates that under direct solar influence this region is sometimes composite. Evidence of the existence of ionization between regions E and F ("intermediate region") has also occasionally been obtained at noon so that the whole ionospheric configuration may be regarded as a composite structure of four components. From the point of view of practical radio communication, however, it is sufficient to regard the ionosphere as divisible into two main regions, E and F, but where it is necessary to consider further the ionospheric fine structure it is proposed to name the four components, at successive increasing equivalent heights, regions E^{I}, E^{II}, F^{I} and F^{II}. Usually only regions E^{I} and F^{II} are of importance. (*c*) The rate of decay of region-F ionization during the night has been measured and found not to be explicable in terms of the simple law of recombination between ions and electrons. (*d*) Measurements have been made of the rate of production of electrons in region F during the period following sunrise.

§ 1. INTRODUCTION

In recent work on the exploration of the ionosphere by means of electric waves it has become increasingly clear that one of the most useful indices of ionospheric conditions is a curve exhibiting the equivalent height of reflection as a function of electric wave frequency. In the early experiments of 1925–8 the diurnal variation of the equivalent height of the reflecting region for a single mean frequency was studied, and it was such experiments which led to the recognition of the separate existence of an upper reflecting region situated above the Kennelly-Heaviside layer as well as the phenomenon of magneto-ionic splitting. But it has since been realized that work on a single frequency, useful though it has proved, is not likely to give us the maximum amount of information concerning ionospheric conditions, and that the extra amount of effort required to make hourly series of measurements on as many frequencies as possible is well worth while.

P'

t

f

For measurements made by both frequency-change and group-retardation methods it has been shown that the results in each case give us the value of P', the equivalent path of the atmospheric or space-waves. The type of record most frequently obtained in ionospheric observations is therefore that exhibiting the relation between P' and the time t. I shall refer to this as a (P', t) curve. There have been described recently many elegant methods by which this type of curve can be obtained by automatic registration, and various groups of observers are obtaining practically continuous records of this kind. It is my object in this paper, however, to emphasize the advantages of supplementing such observations by measurements made on as many frequencies as possible, and to discuss briefly two methods which have been found useful in this connexion. If we make, as nearly simultaneously as possible, a series of measurements of P' for a wide range of mean frequency f we can draw a (P', f) curve. From this it is possible to deduce the maximum electronic content of both lower and upper regions and so study the diurnal and seasonal variation of ionization. No such corresponding amount of information can be obtained from a (P', t) curve. The determination of (P', f) data with as close a frequency-spacing as possible is therefore one of the two methods advocated and discussed below. The second method is a compromise between the (P', t) and (P', f) series of measurements and will be referred to as the (P', t, f) method. With our present technique its prosecution is a less arduous matter than the rapid measurements required for hourly (P', f) curves, but it has the disadvantage that it yields information concerning the ionization in only one or other of the two reflecting regions.

§ 2. THE DETERMINATION OF (P', f) DATA

In 1930⁽¹⁾ it was pointed out that by measuring the critical penetration frequency for the Kennelly-Heaviside layer at different times of the day the diurnal variation of the maximum ionization content could be studied. An experimental study of the ionization of the lower region was made on these lines with (P', f) data obtained by the frequency-change method. In this case the values of f were spaced 0·25 Mc. sec. apart. On changing over to the group-retardation method at King's College in 1931, it was possible to make measurements more rapidly in time and with closer frequency-spacing and these led to the recognition of the dependence of the critical penetration frequency on the polarization of the waves. In still more recent work at the Radio Research Station, Slough, and at the Halley Stewart Laboratory, King's College, London, working in conjunction with the National Physical Laboratory, improved technique has permitted the determination of (P', f) data on frequencies 0·05 to 0·1 Mc./sec. apart, the time spent on a single frequency being about 15 sec. But even this procedure is not ideal, and it is hoped that, before long, apparatus will be developed which will yield automatically (P', f) curves comparable with the automatic records of (P', t) which are now being obtained. If, at the same time, the receiver could be made to respond either to left-handed or to right-handed circularly polarized downcoming waves the results would be still further enhanced in value.

§ 3. THE CALCULATION OF MAXIMUM IONIZATION CONTENT FROM CRITICAL FREQUENCY DETERMINATIONS

As the experiments we are considering are carried out with only a small distance between sender and receiver it is assumed that the waves are incident normally on the ionized region which reflects them, and it is assumed further that reflection takes place at a level at which the refractive index μ of the medium is reduced to zero by the presence of electrons or ions. It is recognized that the second of these assumptions is only an approximation, since it is just for the conditions in which μ is small compared with unity that the simple treatment of wireless wave propagation in terms of rays breaks down, for it is not then possible to consider the properties of the medium as remaining sensibly constant within the dimensions of a wave length in the medium. It is, however, clear from the accurate wave treatment of Hartree that, as soon as the square of the refractive index becomes slightly negative, marked reflection takes place even if the change of properties with height is not abrupt, so that, until the subject is further developed on the theoretical side, the best we can do seems to be to consider the condition $\mu^2 = 0$ as the criterion for reflection at normal incidence. Although it is most probable that further progress on the theoretical side will involve a revision of the actual magnitudes of the maximum ionization content it is unlikely that it will necessitate a marked revision of the deduced diurnal trend of this quantity.

As I have shown (2) for a magneto-optical medium such as the upper atmosphere, the condition $\mu^2 = 0$ is reached for any direction of propagation relative to the earth's magnetic field, when, approximately, either

$$N = \frac{3}{2}\frac{\pi m}{e^2} f^2 \qquad \ldots\ldots(1),$$

or when

$$N = \frac{3}{2}\frac{\pi m}{e^2} (f^2 \pm f f_H) \qquad \ldots\ldots(2),$$

where N is the number of electrons (of charge e and mass m) per cm.^{3}, f is the frequency of the waves and f_H is equal to 1·32 Mc./sec., the characteristic gyro-frequency of electrons under the influence of the earth's magnetic field. If, therefore, f represents a critical frequency, N represents a maximum ionization content. Which of these formulae is applicable in any case depends on various factors. For the extraordinary ray, which is of right-handed circular polarization for the conditions we are considering, the relevant formula is (2) with the lower sign. For the ordinary ray, however, whether we use (1) or (2) with the upper sign depends on the magnitude of a certain critical ratio $2p_L \nu / p_T^2$ where p_L and p_T are the natural angular frequencies of rotation of an electron about the longitudinal and transverse components of the earth's magnetic field and ν is the frequency of electron collisions with the air molecules. If this ratio is large compared with unity the propagation resembles the longitudinal type and the appropriate formula for the ordinary wave

N, e, m, f

p_L, p_T

ν

is (2) with the upper sign. If, on the other hand, this ratio is small compared with unity the propagation resembles the transverse type and the appropriate formula is then (1). It seems fairly certain that for the upper region, where ν is small, $2p_L \nu / p_T^2$ is also small, so that we have

$$N = \frac{3}{2} \frac{\pi m}{e^2} f^2 \text{ ... (ordinary ray)} \qquad \text{......(3)}$$

and

$$N = \frac{3}{2} \frac{\pi m}{e^2} (f^2 - f f_H) \text{ ... (extraordinary ray)} \qquad \text{......(4).}$$

It may be noted that for constant N the critical conditions may be simultaneously reached for an ordinary wave frequency of, say, f_1 and for an extraordinary wave of frequency f_2. The relation between f_1 and f_2 in this case is, evidently,

$$f_1^2 = f_2^2 - f_2 f_H \qquad \text{......(5),}$$

or

$$f_2 - f_1 = \frac{f_2}{f_2 + f_1} f_H \qquad \text{......(6).}$$

Now if f_1 and f_2 are of the order of 4 to 6 Mc. sec. as is the case in practice for the upper region, while f_H, which is determined by the magnitude of the earth's total force, is 1·32 Mc./sec., the value of $(f_2 - f_1)$, according to (6), ranges from 0·66 to 0·68 Mc./sec. We shall see later that experimental values of this order of magnitude are obtained.

For the lower ionized region where the critical conditions at noon are reached at 3 to 4 Mc./sec. according to season, the corresponding value of $(f_2 - f_1)$ ranges from 0·78 to 0·68 Mc./sec. if the conditions approximate to the transverse type of propagation. If, however, the collisional frequency is sufficiently high to make the ratio $2p_L \nu / p_T^2$ much greater than unity the propagation would approximate more closely to the longitudinal type and we should then have for the ordinary wave of frequency f_1

$$N = \frac{3}{2} \frac{\pi m}{e^2} (f_1^2 + f_1 f_H) \qquad \text{......(7)*,}$$

and, since, as before we have for the extraordinary wave

$$N = \frac{3}{2} \frac{\pi m}{e^2} (f_2^2 - f_2 f_H) \qquad \text{......(8),}$$

we also have

$$f_2 - f_1 = f_H \qquad \text{......(9).}$$

Summarizing, therefore, we may expect, according to conditions, values of $(f_2 - f_1)$ ranging from 0·66 to 1·32 Mc./sec.

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* Equations (1) to (4), (7), (8) and (10) have been derived from the theory of dispersion with the inclusion of the term in the expression for the refractive index which, according to Hartree, expresses the discreteness of the scattering centres. If this term is neglected, since its inclusion is still a debatable matter, the expressions for N must all be multiplied by $\frac{2}{3}$.

§ 4. THE INTERPRETATION OF (P',f) DATA WITH SOME ILLUSTRATIVE EXAMPLES

At the outset it may be stated that, for the conditions we are considering, the influence of the earth's magnetic field is such that we are obliged to take into account the polarization of the returned waves, and, in fact, consider the (P', f) curves for the ordinary wave and for the extraordinary wave separately. It is found, for reflection from both regions E and F, that there are very frequently obtained echo doublets and it is rarely a matter of difficulty to decide which is the extraordinary pulse and which is the ordinary one. This interpretation follows on the lines previously discussed by Appleton and Builder(3). For conditions in which electron limitation is not operative, the ordinary ray is less absorbed, due to collisional friction, than is the extraordinary ray. This may be predicted from theory for the range of frequencies we are considering and is found to be the case in practice. If, moreover, the records of echo-delineation are made with a time base which is locked to the electrical mains which control the pulse-frequency of the sender, it is possible to photograph the echo patterns for the different frequencies successively on the same record, so that the ground reference pulses are immediately under one another in a straight line. From such a record the tracing of the separate (P', f) curves for the ordinary and extraordinary rays is a simple matter. When the successive photographs have been obtained with small frequency spacing one can, on looking along the record, see immediately the different run of the two curves*.

For the upper region the importance of magneto-ionic double refraction is now well established. Until recently there was some doubt concerning its influence in causing echo-doubling in reflection from the lower region, although it was known that characteristic polarizations due to differential absorption were thereby produced. This doubt has been recently removed by experiments carried out in this laboratory by Mr F. W. G. White, who has shown that magneto-ionic splitting is present for region-E reflection as in the case of region-F reflection, but that, owing to marked absorption, the extraordinary ray is usually very weak in intensity. In some of the data discussed below examples of E-region splitting can be clearly recognized, confirming the results of Mr White†.

Before we proceed to discuss typical (P', f) curves it may be well to consider the type of curve to be expected if there are in the ionosphere, at any time, two main regions of electrification, the upper region being more strongly ionized than the lower. For the lower frequencies we should expect the reflection of both components from the lower region, the extraordinary ray being strongly absorbed. With increasing frequency the ordinary ray will be the first to suffer electron limitation and so, penetrating region E, will be reflected at region F. With increasing frequency the extraordinary ray will also eventually penetrate region E and then both rays will be reflected from the upper region. The two types of magneto-ionic splitting described by Appleton and Builder as "group-retardation splitting" and "stratification splitting" may then be expected to occur in the order named, until with still higher frequencies the penetration of the upper region by the ordinary and extraordinary rays takes place in succession. The examples taken for discussion below are all found to be broadly explicable on such lines but in some cases illustrate also phenomena not previously described.

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* For a description of the apparatus used see *The Cathode-ray Oscillograph in Radio Research*, by Watt, Herd and Bainbridge-Bell (London, 1933).

† As has previously been pointed out(4), the difference between the absorption of the extraordinary and ordinary waves is most marked for the critical gyro-frequency 1·32 Mc./sec., so that for frequencies about this value only the ordinary ray reaches the ground in appreciable intensity. The results of Meissner(5), who found no anomalies for a wave length of 212 metres, are thus explained

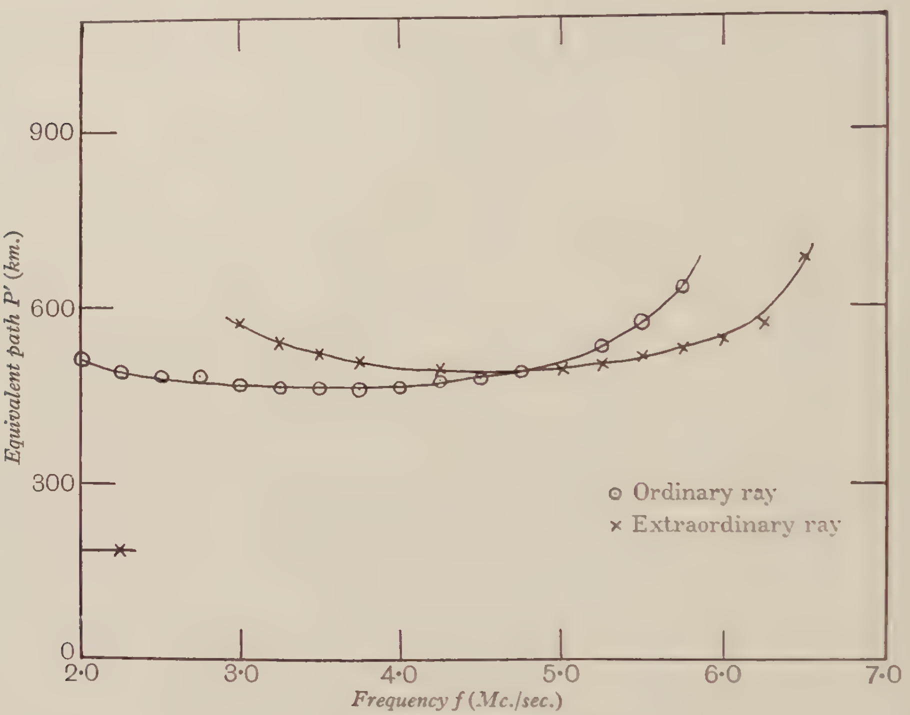

Figure 1. (P', f) curve for 1600 G.m.t., October 19, 1932.

Example 1. In figure 1 is shown the (P', f) curve for conditions at 1600 G.m.t. on Oct. 19, 1932, at the Radio Research Station, Slough. This figure clearly illustrates the difference in the curves for the ordinary and extraordinary rays. For a frequency of 2·0 Mc./sec. marked reflection from region F was found, reflection from region E having practically ceased, but the frequency had to be increased to 3·0 Mc./sec. before marked reflection of the extraordinary ray from region F took place. Although low echo-amplitude, due no doubt to absorption, prevented an accurate determination of the critical frequencies f_1 and f_2 for region F, the general trend of the curves shows that the quantity ($f_2 - f_1$) was about 0·66 Mc./sec., in agreement with equation (6) above for the quasi-transverse type of propagation. It will be seen that both group-retardation splitting and stratification splitting are well illustrated in figure 1.

Example 2. In figure 2 is shown the (P', f) curve for conditions at noon on April 10, 1933, at the Radio Research Station, Slough. Here the region-E splitting is clearly shown. At about 3·0 Mc./sec. the change-over from reflection at region E to reflection at region F took place. At first the increase of equivalent path with increase of frequency for reflection at region F was not marked. With a further increase of frequency, however, greater penetration of the upper region took place and P' reached a maximum at 4·2 Mc./sec. (The significance of this maximum is further discussed below.) The fall in P' with further increase of frequency suggests another group-retardation effect. Ultimately the ordinary ray penetrated region F at 5·6 Mc./sec.

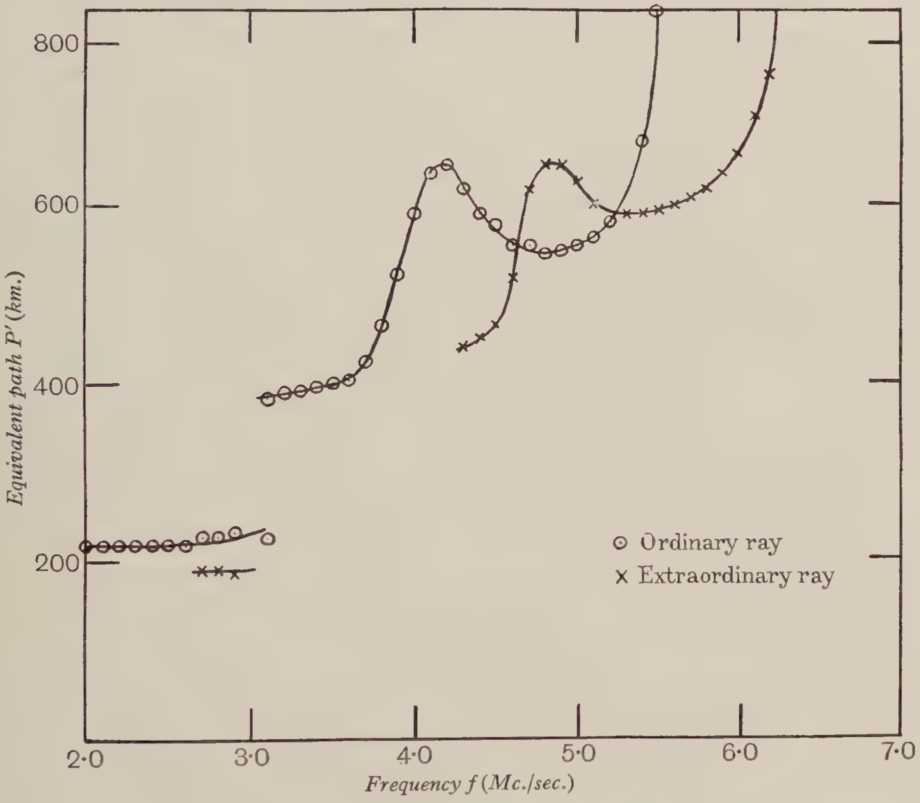

Figure 2. (P', f) curve for 1200 G.m.t., April 10, 1933.

It will be seen that the curve for the extraordinary ray has quite a similar trend but is displaced 0·70 Mc./sec. on the frequency scale. This difference is approximately that mentioned above for the difference $(f_2 - f_1)$ between the critical penetration frequencies for quasi-transverse propagation. That the same frequency-spacing is maintained for frequencies lower than these penetration values is no doubt due to the fact that, in the range of frequencies we are considering, the transmission characteristics of an ordinary ray of any frequency f_0 Mc./sec. are not markedly different from those of an extraordinary ray of frequency $(f_0 + 0·66)$ Mc./sec. To calculate

$N_{max.}$ for region F from the critical penetration frequency for the ordinary wave equation (3) is the appropriate formula, while in the case of the extraordinary wave equation (4) should be used. It is clear from the critical frequency-spacing that the values obtained in either case would be approximately the same.

The maximum in the (P', f) curve for region F is a phenomenon not previously recorded and clearly indicates a variation in ionization with height different from that calculated on the assumption of a single monochromatic ionizing radiation acting on the earth's atmosphere. The fact that for frequencies higher than 4·3 Mc./sec. the equivalent height falls with increasing frequency indicates that there must be a sudden increase of ionization with height at a particular value of ionization. At present it is not quite certain what is the cause of this region-F ledge, as it may be called, and it would be a matter of interest to test, at the next solar eclipse, whether it is due to ultra-violet light or to neutral particles. From a study of this maximum extending over some months it is now possible to make the following statements concerning it: (*a*) it is not apparent at night; (*b*) it is more marked in summer than in winter; (*c*) the frequency for which it occurs increases with the general ionization in region F caused by the sun. As an example of this the following may be quoted. On May 9, 1933, at the following times the maximum occurred at the frequencies (Mc./sec.) mentioned in brackets: 0700 (3·8); 0800 (4·0); 1000 (4·3); 1200 (4·5); 1400 (4·3); 1600 (4·1); 1900 (maximum not detectable).

It is not improbable that this effect is the same as that mentioned by T. L. Eckersley in his discussion(6) of short-wave transmission phenomena as elucidated by the Marconi facsimile method. Eckersley found that his observed values for the equivalent height of reflection from region F were ranged in two series depending on the wave length used, and suggested that the upper region might be composite. It is difficult, however, to understand this author's statement that the lower-frequency waves would be reflected at the sharply defined upper portion of region F while the higher-frequency waves would be reflected at the lower part.

The case of the penetration of region E in figure 2 is not so easy to unravel. It is clear that the ordinary-ray penetration occurred at 3·0 Mc./sec., but, owing to the low extraordinary-ray amplitude caused by absorption, there is a range of frequencies in which the echo-amplitude is not appreciable. It is thus not possible here to determine f_2 for the lower region, so that, $(f_2 - f_1)$ being uncertain, it is not possible to make a definite decision in choosing between the two alternative formulae for calculating $N_{max.}$.

Example 3. In figure 3 is shown the (P', f) curve obtained at noon on April 8, 1933, at the Radio Research Station, Slough. This curve is of special interest as showing reflection from equivalent heights between region E and region F. Many measurements made within the last few years have forced on us the necessity of recognizing that reflection can, on occasion, take place at such an *intermediate region*, as it may be called. Such a region is detected rarely at noon, but more frequently in the early morning.

In this figure is clearly seen the effect of splitting in region E. The fall in P' with increase of frequency for the intermediate region indicates that there is a discontinuous jump here in the case of the ordinary-ray curve. It is clear that the intermediate region is not the same as the region-F ledge, for the maximum indicating the influence of the latter is found at a much higher frequency.

Example 4. The (P', f) data exhibited in figures 4 and 5 may be conveniently considered together, since they were obtained on the same day, May 9, 1933, at the Radio Research Station, Slough. Figure 4 represents conditions at 0800 G.m.t. and is of interest as showing very clearly the splitting in region E*. Moreover, the magnitude of the frequency separation between the ordinary and extraordinary curves here strongly suggests quasi-transverse conditions. In figure 4 the shelf maximum in region F is not very marked since it is an early morning curve.

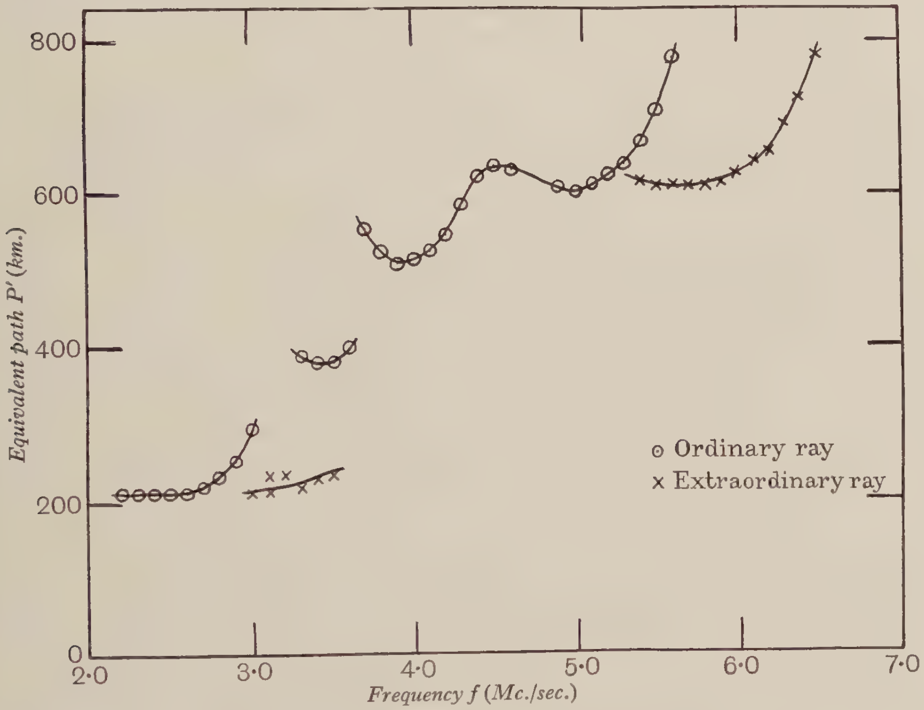

Figure 3. (P', f) curve at 1200 G.m.t., April 8, 1933.

In figure 5, which represents conditions on the same day at 1000 G.m.t., we note that again splitting is marked in region E. Also the ordinary-ray curve is seen to include values for P' almost up to region-F values. Although part of these abnormally high equivalent heights may be regarded as due to group-retardation effects (we may compare in this connexion the high values for P' for frequencies

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* A word of caution is needed here, since double echoes from region E are often noted which do not consist of magneto-ionic components. Sometimes we have simultaneous reflection from regions EI and EII, while sometimes we have simultaneous reflection from region E caused by ultra-violet light and from the ionized region produced, at a slightly lower height, by abnormal ionizing agencies, e.g. corpuscles. Such abnormal ionization has been shown by Mr Naismith and the writer, *Proc. R. S.* A, **137**, 36 (1932), and *Proc. Phys. Soc.* **45**, 389 (1933), to be correlated either with magnetic storms or with thunderstorms.

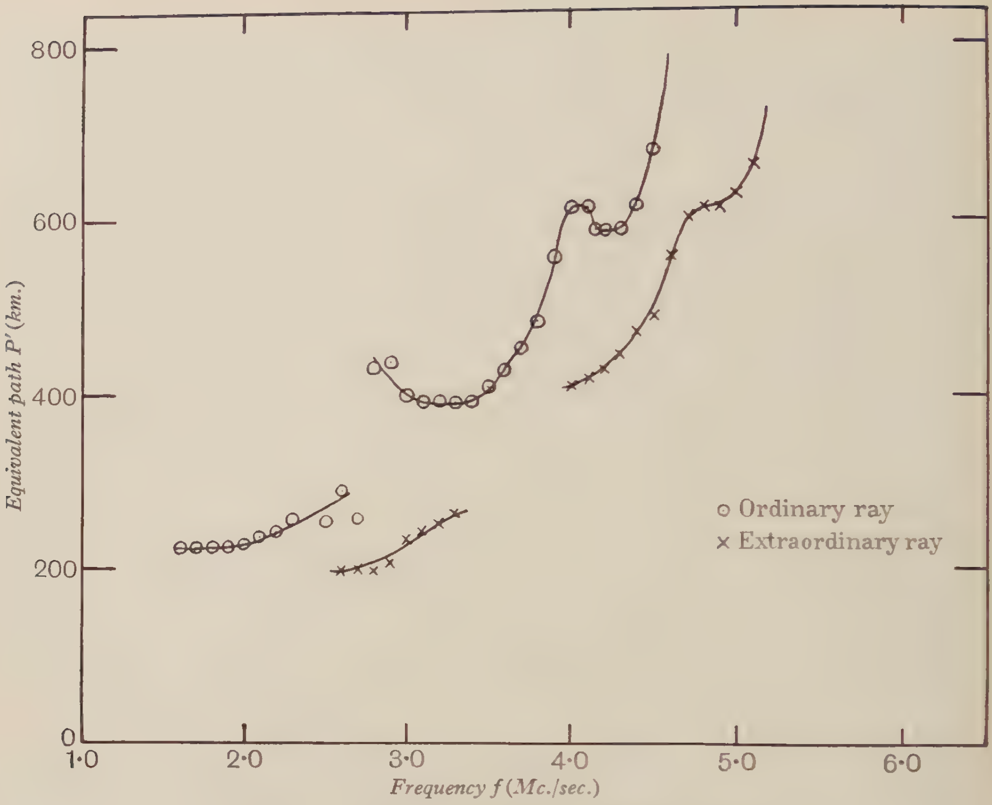

Figure 4. (P′, f) curve at 0800 G.m.t., May 9, 1933.

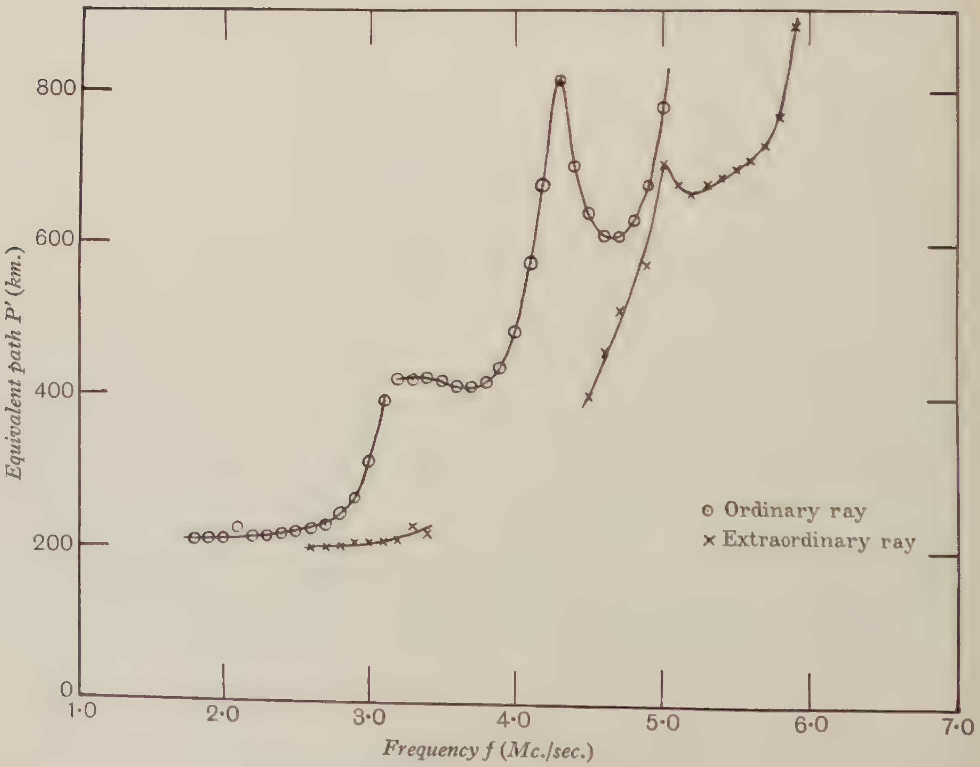

Figure 5. (P′, f) curve at 1000 G.m.t., May 9, 1933.

approaching the region-F penetration values), it is unlikely that the whole effect is due to this cause and we must conclude that there is a considerable amount of ionization between the two main regions. Further reference will be made to this point in the discussion below.

For the region-F parts of the curves the shelf-maximum has become more pronounced as noon is more nearly approached. Moreover, at 1000 G.m.t. it now occurs for the ordinary ray at 4·3 Mc./sec. whereas two hours earlier (see figure 4) it occurred at 4·05 Mc./sec.

§ 5. FURTHER DISCUSSION OF THE ILLUSTRATIVE EXAMPLES

From the data briefly summarized above it is clear that we must consider the ionosphere as being made up of at least four components. As the data refer to days free from anomalous effects due to magnetic disturbances it is clear that these four components are produced by normal solar radiation. Moreover, the general dependence of the electron contents on solar altitude also indicates that the radiation travels in straight lines. It is therefore considered most probable that ultra-violet light is the radiation in question.

The existence of considerable ionization between the two main regions E and F during the daytime has been suspected for a considerable time now, but since, at noon, it is normally less than that in region E it cannot usually be investigated by the methods we are considering. In the early morning hours, however, the sun's rays ionize successively lower strata which have been largely depleted of ionization during the night, so that the intermediate region becomes ionized before region E has been sufficiently ionized to mask it. Reflection in succession from region F, the intermediate region, and region E is therefore frequently observed in the morning*. On the exceptional day of April 8, 1933, however, the maximum ionization in the intermediate region was greater than that of region E about noon, and from the critical penetration frequencies read from the (P', f) curve the following maximum ionization-densities (in electrons per cm.^{3}) may be deduced: region E ($1{\cdot}8 \times 10^5$); intermediate region ($2{\cdot}5 \times 10^5$); region-F ledge ($3{\cdot}8 \times 10^5$) and main region F ($6{\cdot}1 \times 10^5$).

The existence of the region-F ledge is inferred from the fall in equivalent height with increase of frequency—see, for example, figure 2. The frequency at which the maximum occurs on the region-F portion of the (P', f) curve may be taken as being approximately the penetration frequency of the ledge. Waves of frequency immediately higher than this value do not reach much greater heights, and the reduction of P' with increase of f is due to the reduction in the group-retardation effect of the ledge itself. For the conditions indicated by figure 2 the electron-distribution with height must be somewhat as shown in figure 6, where the relative values of the ordinates are approximately correct but not those of the abscissae.

Concerning the question of nomenclature, since it is still clearly necessary to retain the division into the two main reflecting regions E and F, it is suggested that the four components should be designated as shown in this diagram. In this connexion it is convenient to retain capital-letter symbols to designate regions caused by normal solar influence and to use small-letter symbols for regions caused by abnormal agencies.

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* J. A. Ratcliffe and E. L. C. White have obtained independent evidence of this using their method of automatic registration of layer heights.

Such a composite structure as I have pictured for the ionosphere is not considered unlikely when it is remembered that Pannekoek(7) has shown that the level of the maximum ionization caused by ultra-violet light depends on the ionization potential of the gaseous constituent in question. We might therefore expect a maximum of ionization for each constituent, atomic or molecular. As, however, there is laboratory evidence that ionization can be produced by photons of energy less than that corresponding to the ionization potential, it is probable that the matter is further complicated by the processes of cumulative ionization.

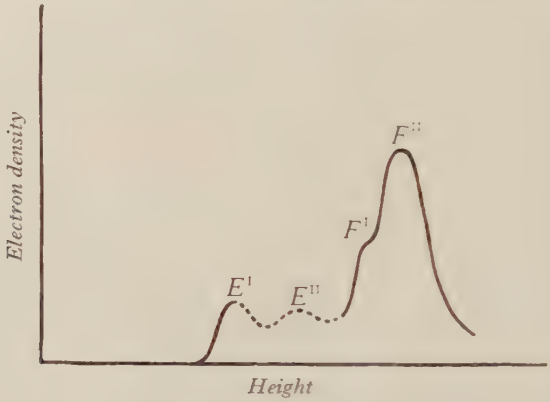

Figure 6.

§6. THE DETERMINATION AND INTERPRETATION OF (P', t, f) DATA

As has previously been mentioned the (P', t, f) method is a compromise between the methods involving the determination of (P', t) and (P', f) data. It requires a certain amount of prevision on the part of the observer in that he has to anticipate whether the ionization is increasing or decreasing. For the case of the normal ionization produced by the ultra-violet light from the sun there are now sufficient data to guide one in this respect, but the weakness of the method is that it is difficult to prosecute during conditions of abnormal ionization.

Let us consider a simple normal case as an illustration. During the night, from about midnight till about an hour before sunrise, it is usual for the ionization in the upper region to decrease steadily. Let us suppose that at the beginning of this period we work with a frequency which is just less than the critical penetration value for the extraordinary wave. (For such conditions the ordinary wave will be penetrating the region in question.) If a (P', t) curve is obtained, by automatic registration or otherwise, the usual gradual increase of P' with t will be noticed and recorded and eventually there will be no appreciable reflection. The frequency can then be changed to a slightly lower value (0·2 Mc./sec. steps are convenient) and the process repeated. In the case of increasing ionization, as in the early morning period, the corresponding steps are, of course, increases in frequency. From the examination of the record we obtain in this way the times at which extraordinary waves of different frequencies penetrate the region.

An example of the type of record obtained in this way is shown in figure 7, where the equivalent height was automatically registered during the three hours 0000 to 0300 G.m.t. on April 26, 1933, in an experiment carried out at the Radio Research Station, Slough. The going out of the extraordinary wave for the five frequencies 4·0, 3·8, 3·6, 3·4 and 3·3 Mc./sec. in order is seen to occur at the following times: 0009, 0051, 0126, 0215 and 0248 G.m.t.

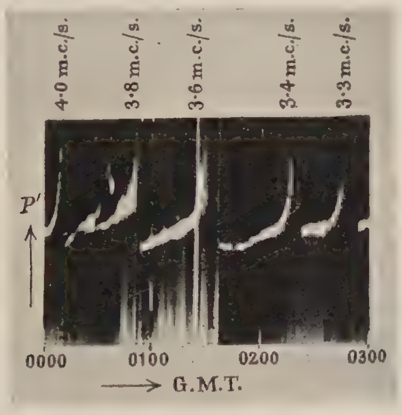

Figure 7. Relation between equivalent path P' and time on April 26, 1933.

To calculate the maximum electronic content at these times we insert the appropriate frequencies in the formula

$$N = \frac{3}{2}\frac{\pi m}{e^2}(f^2 - ff_H) = 1{\cdot}86 \times 10^{-8}(f^2 - ff_H) \qquad \text{......(10),}$$

where f_H is 1·32 Mc./sec.

We are thus able to plot a curve illustrating the variation of $N_{\text{max.}}$ with time. In figure 8 are shown the results of the run of which figure 7 is an illustrative record. Here the ionization is seen to be decreasing rapidly at first, but, at 2230, the rate of decrease is much reduced and continues to remain so throughout the greater part of the night. This reduction in the rate of the disappearance of the ionization is not understood, unless it be due to the fact that the height at which the maximum ionization exists increases with time (as, indeed, the records show that it does), so that the pressure and thus the magnitude of the recombination or attachment processes decrease correspondingly. The ionization is seen to begin to increase at least 45 minutes before ground sunrise, a phenomenon which is to be expected in a region at a considerable height above the earth's surface.

It is not proposed to consider here the full significance of the information depicted in figure 8, as obviously at least a full year's data of this type would be required for adequate discussion. One or two points, however, may be mentioned. In examining the decay of ionization during the period 2230 to 0315, for which period the curve suggests steady conditions, we are naturally led to enquire how far the dissipative process conforms with the usually accepted law of recombination which is expressed by the formula

$$\frac{1}{N}-\frac{1}{N_0}=\alpha t \qquad \text{......(11)},$$

N_0, N where N_0 is the electron-concentration at zero time and N the concentration at any
t subsequent time t. On plotting $(N^{-1}-N_0^{-1})$ against t, however, we obtain a curve

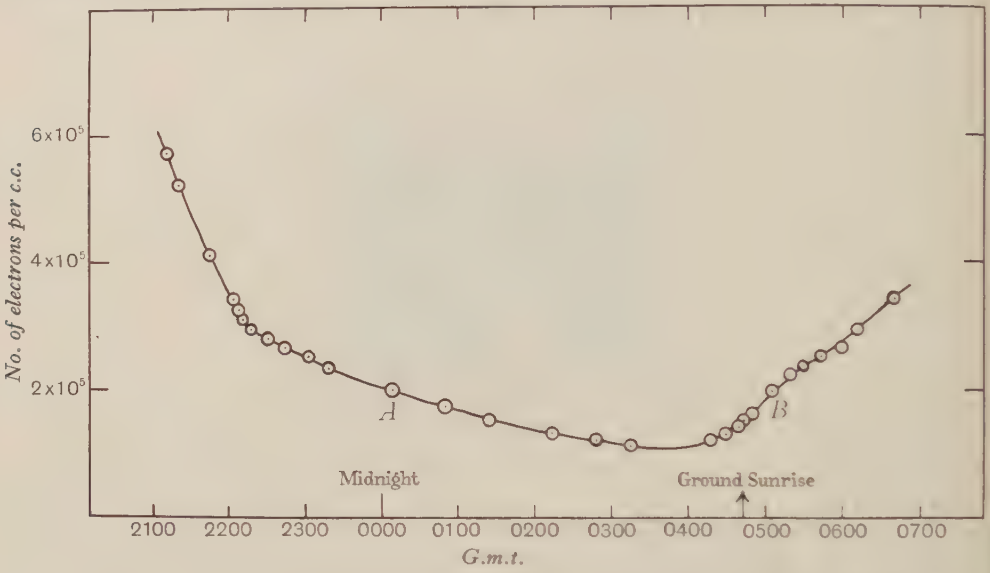

Figure 8. Curve showing variation of ionization in upper region, April 25–26, 1933.

which is not straight but definitely concave upwards, indicating an increase of α with time. This would indicate, contrary to the conclusions of Eckersley(8), that equation (11) is not strictly valid during this period. If, however, we choose to represent approximately the sequence of events in terms of recombination according to equation (11) we may remark that the numerical value of α ranges from $2{\cdot}5 \times 10^{-10}$ to $4{\cdot}0 \times 10^{-10}$.

There is, however, another process which may gradually render the electrons ineffective during the night, namely, their attachment to neutral atoms. In this case we should expect the following relation to hold in place of equation (11):

$$\log N_0 - \log N = \beta t \qquad \text{......(12)},$$

β where β is the attachment coefficient. Here a plot of log N with t gives a curve which is fairly straight and a numerical value for β of $6{\cdot}0 \times 10^{-5}$. The matter cannot, however, be regarded as settled and is reserved for a later discussion.

From a curve such as that shown in figure 8 data concerning the rate of production of ionization may be deduced. Consider a point on the falling part of the curve, for instance A. Then the slope of the curve $(dN/dt)_A$ represents the rate at which the electrons are disappearing. Now at a corresponding point B on the rising part of the curve where the ionization is the same, the rate of increase of ionization $(dN/dt)_B$ is determined by two factors, (i) the electron dissipative process, and (ii) the rate of production q of electrons. Since we may assume that for similar magnitudes of ionization the rate of disappearance of electrons is the same we see that q is equal to the numerical sum of $(dN/dt)_A$ and $(dN/dt)_B$.

Making use of such theoretical considerations for the data illustrated in figure 8 we find that the rate of electron-production increases with time as shown in the table.

G.m.t. April 26, 1933	0345	0421	0446	0530	0616
q (electrons per cm^3 per sec.)	0	21	30	38	55

§7. ACKNOWLEDGMENTS

The work described in this paper was carried out as part of the programme of the Radio Research Board of the Department of Scientific and Industrial Research. I am indebted to Mr R. A. Watson Watt and Mr J. F. Herd for providing facilities for its prosecution at the Radio Research Station, Slough, and also to them and to my collaborator, Mr R. Naismith, for much helpful discussion. I also wish to thank Mr G. H. Calver for much valued assistance in making the measurements.

Note added June 6. When this paper was ready for publication a most interesting letter by Messrs Schafer and Goodall appeared in the issue of *Nature* for June 3, 1933, in which very similar results concerning the structure of the ionosphere are announced as a result of their observations in America. The opportunity of comparing the full account of their methods and data with that given in the present paper is awaited with much interest.

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DISCUSSION

Mr A. O. Rankine. I would like to ask Prof. Appleton to explain the use of the terms "ordinary" and "extraordinary" in relation to the rays discovered in his remarkable experiments. He has spoken of the two reflected beams as being circularly polarized in opposite senses, a phenomenon which in optics is associated with rotation of the plane of polarization in optically active substances or in other transparent materials stimulated by a magnetic field. These two rays, discovered and explained by Fresnel, are not usually called "ordinary" and "extraordinary," although in the special case of quartz, which displays optical activity along its axis as well as birefringence in other directions, it is clear that the two effects are closely interlinked. Is the effect of the earth's magnetic field on the ionosphere simply that of imparted optical activity—the Faraday effect—or does it produce double refraction in the generally accepted sense as well?

Author's reply. The influence of the earth's magnetic field on the ionosphere is similar to that with which we are familiar in the case of the Faraday effect where, as Prof. Rankine expresses it, optical activity is imparted to the medium by the imposed magnetic field. Rotation of the plane of polarization occurs for propagation along the direction of the magnetic field and double refraction for propagation at right angles. Of the two component waves, one is little influenced by the action of the field while the other is much influenced and exhibits anomalous-dispersion phenomena at the natural gyro-frequency. These two components have (most probably without justification, I fear) been respectively termed the "ordinary wave" and the "extraordinary wave."

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STUDIES IN THE EXTREME ULTRAVIOLET AND THE VERY SOFT X-RAY REGION

The Eighteenth Guthrie Lecture, delivered on June 16, 1933,

BY PROF. M. SIEGBAHN

§ 1. INTRODUCTION

ONLY a few years ago X-ray spectra and optical spectra were separated by a rather large gap, extending over 3 or 4 octaves of wave-lengths. The spectroscopic methods used at that time for studying these two ranges were qualitatively different. In the X-ray region, crystal lattices were the only effective means for the analysis of the radiation. The fundamental formula

$$n\lambda = 2d . \sin \phi$$

shows that the method is limited to values of λ which are smaller than $2d$, where d is the distance between the atomic layers serving as reflecting planes. In practice this means that the crystal lattice method is limited to wave-lengths below, say, 20 Å.

On the other hand it had been shown by Millikan that the ruled grating used in the manner adopted at that time could register optical spectra of wave-lengths down to, say, 200 or 300 Å.; in a few cases some shorter wave-lengths had been observed.

A new era of the spectroscopy was begun when advantage was taken of the experimental fact that radiation of short wave-length was strongly reflected at grazing incidence. By using plane and concave ruled gratings it was thus possible at least in principle to cover all the formerly unknown region between the X-ray and extreme ultraviolet regions. Experimentally there were many problems to be solved before the expected results could be gained. It will be the object of this lecture to give a preliminary report on some of the work done on these problems at the laboratory of Upsala during the past year.

§ 2. THE USE OF RULED GRATINGS IN THE EXTREME ULTRAVIOLET AND THE VERY SOFT X-RAY REGION

Figure 1 illustrates the effects obtained with concave and plane gratings. In the earlier experiments for registering the short-wave-length region the radiation was reflected at incidence nearly normal to the concave grating as at (I). In the new methods (II, III) the incoming radiation strikes the grating at an angle of only a few degrees. From the geometry of the concave grating it is immediately seen that this also means that the plate, which must be bent to the Rowland circle, receives the radiation at a very small angle as at (II). This gives rise to an experimental difficulty which increases greatly as soon as one tries to diminish further the angle of incidence, as is necessary for registering the shortest wave-lengths.

In this respect the plane grating (III) has the advantage that the plate is placed nearly normally to the radiation. But to get well-defined lines in this case it is necessary to cut out a small pencil of radiation by means either of two narrow slits or of one slit used with a very small grating. Both these methods have been used. Up to this time practically all results in the very soft X-ray region have been achieved by the plane-grating method.

In comparison with the concave grating, the plane grating has decided disadvantages which make it desirable to try to apply the former to this region also. It is to be remembered that the pencils of rays at the plane grating are always divergent, so that sharpness of the lines can only be obtained by diminishing the width of the pencil and thereby reducing the number of lines of the grating which are effective. As the resolving-power is determined by this number, the plane-grating method is far inferior to the concave-grating, giving only $\frac{1}{10}$ to $\frac{1}{100}$ of the resolution obtainable with the latter. For these reasons our efforts have been devoted to further development of the concave-grating method with the object of using it for the very soft X-ray region.

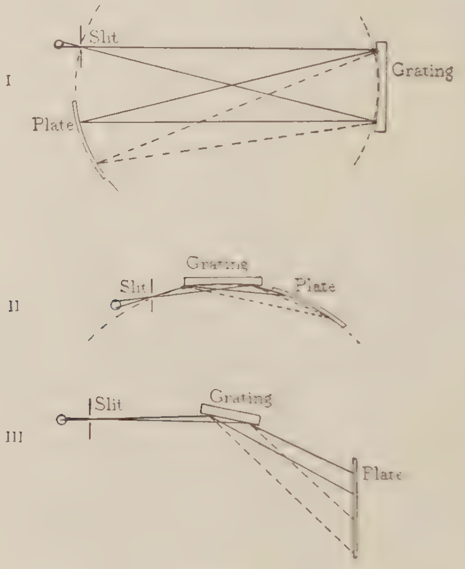

Figure 1.

§ 3. CONCAVE GRATINGS FOR THE VERY SOFT X-RAY REGION

The concave-grating method has been used at the Upsala laboratory during the last 5 years for extensive investigations of the optical spectra in the extreme ultraviolet*. In this case the light-source has been a spark gap in a high vacuum directly connected to the spectrometer and separated from this only by the slit. In some cases it was possible in these experiments to register *optical* spectrum lines with wave-lengths as small as 40 Å.

* By Edlén, Ericsson, Ekefors, Arvidsson, Mack, Söderqvist, Borg and others.

In spite of this, the researches carried out upon very soft X-rays in this region with the same spectroscopic outfit were not successful. During the past year, however, it has yet been possible to find the right experimental conditions for obtaining well-defined lines from X-ray spectra.

One of the main problems in the further development of the concave-grating method with very small angles of grazing incidence was the production of suitable gratings. In particular, it was to be expected that an improvement of the ruling on glass would secure better results, as had been the case in our earlier experiments* with small plane gratings. For this purpose a ruling-engine was constructed and built in the workshop of the laboratory during the past year. Special care was devoted to the ruling-mechanism and the technique of ruling. The gratings which have been ruled on the new machine show an absolutely smooth surface, the lines having the same appearance at the beginning as at the end. The big groove usually found at the beginning of the lines has been entirely eliminated. Gratings up to about 10 cm. may be ruled on the new engine, but the optimum width of the concave gratings used at grazing incidence is considerably less than this, namely 20 to 40 mm.

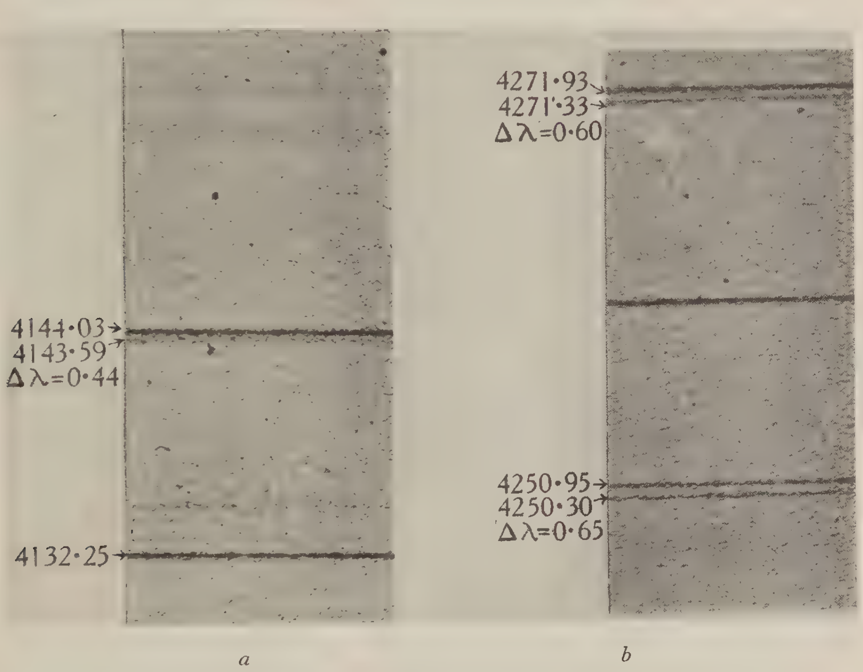

Figure 2.

A test of some of the gratings with ordinary optical light showed that the theoretical resolution was attained. Figures 2 *a* and *b* are reproductions of two spectrograms of narrow doublets from the mercury arc. The grating has about 10,000 lines (576 lines per millimetre) and its radius is about 1·8 m. The ruled surface is about 20 mm. wide, which is the optimum width for a grating used with an angle of 3° to 5° for the incident radiation.

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* Siegbahn and Magnusson: *Z. f. Phys.* **62**, 435 (1930).

Further tests of the gratings were carried out with extreme ultraviolet radiation from optical spectra where the region below 100 Å. was of especial interest. The spark spectra from Al and Cu are reproduced in the figures 3 and 4. The original spectrograms show sharp and well-defined lines down to and below 50 Å.

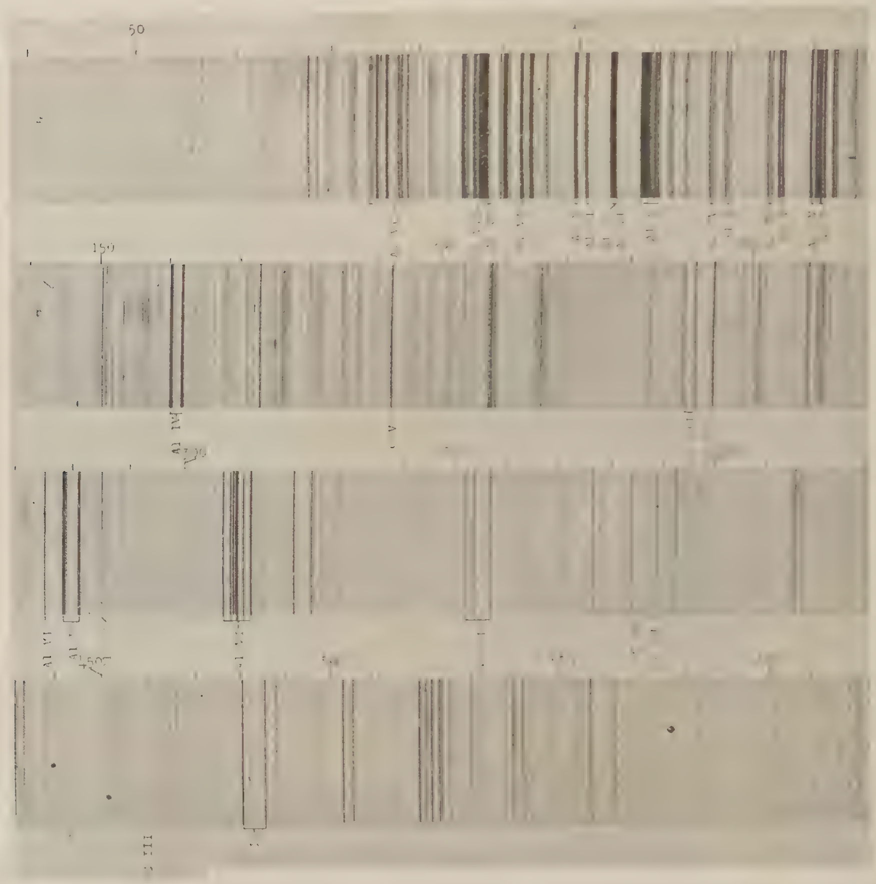

Figure 3. Vacuum spark spectrum of aluminium. 750 sparks, 60 kV., 0·5 μF. Glass-grating T.K. 111, 288 lines/mm., radius 1·8 m., $\phi = 3° 56'$, slit 4μ. Magnification 2·2. Upsala, May 1933.

The enlargements, figure 5, of the Al-groups at 104 Å. in the 4th order are of special interest, as corresponding photographs taken with other gratings have formerly been published*. In spite of the fact that here a grating with only 288 lines per millimetre was used, the resolution is better than formerly and especially the diffuse blackening in the neighbourhood of the lines is reduced so that faint lines near to the strong ones become visible.

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* Ekefors: *Phys. Z.* **31**, 737 (1930). Söderqvist und Edlén: *Z. f. Phys.* **69**, 359 (1931).

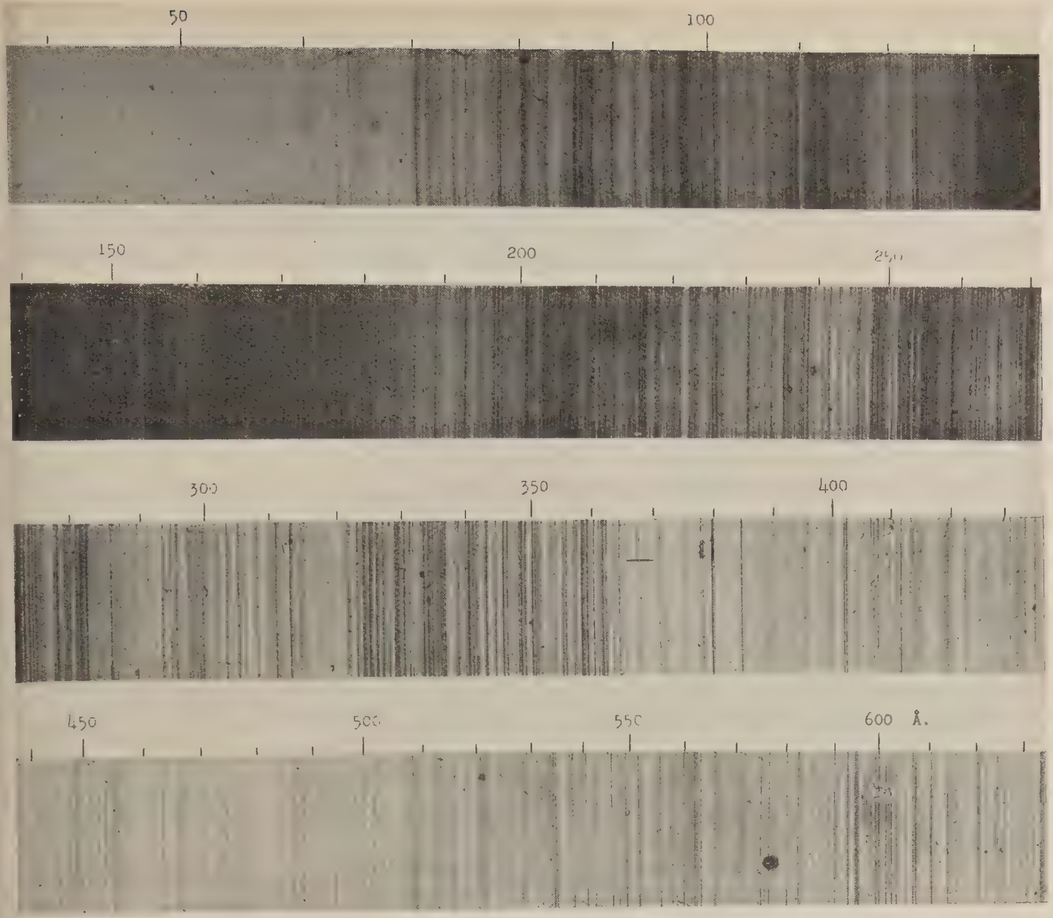

Figure 4. Vacuum spark spectrum of Cu. 750 sparks, 60 kV., 0·5 μF. Glass-grating T.K. 111, 288 lines/mm., radius 1·8 m., $\phi=3^\circ 56'$, slit 4μ. Magnification 2·5. Upsala, April 1933.

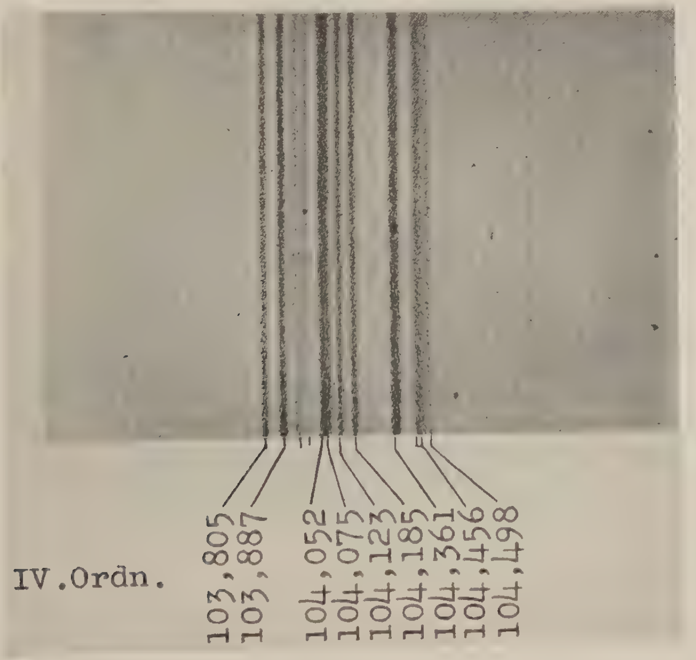

Figure 5 Al groups at 104 Å. in the 4th order. Magnification 25. Grating T.K. 111.

§4. ABSOLUTE MEASUREMENT OF X-RAY WAVE-LENGTHS

Among the problems which have been taken up for experimental investigation with the technique which has been developed I shall first mention the absolute determination of the X-ray wave-lengths, that is to say their values referred to the metric standard. This is of considerable interest as the X-ray measurements carried out with crystals have given us the ratio of these wave-lengths to the dimensions of the crystal lattices, so that a knowledge of the wave-lengths in absolute values will also make possible a direct computation of the atomic distances in absolute values.

In a note published in *Nature* Mr Söderman and I showed a spectrogram taken with a plane grating in which two spectra, one from an X-ray source and the other from a spark, were taken side by side with identical arrangement of the slit, grating and plate. As the optical spectrum contained a number of spectral lines whose wave-lengths were known in absolute values, the X-ray lines could be determined on the same scale.

Owing to the relatively small resolution and dispersion obtainable with plane gratings the accuracy in this determination is not very large. The use of the concave-grating technique now at hand will allow of a considerably higher accuracy. These investigations are now in progress and will soon be published by Mr Söderman.

§5. THE STRUCTURE OF THE X-RAY LINES AND THEIR DEPENDENCE ON THE CHEMICAL AND PHYSICAL STATE OF THE EMITTING ATOMS

The higher resolution and dispersion given by the concave grating makes it possible to resolve and study the structure of lines which with the plane grating appear to be single. In the case of the carbon K_α line, which has been studied by several investigators, the plate reproduced in figure 6 and showing side by side a spark spectrum (aluminium) with known lines and the carbon K_α line permit of an accurate determination of the wave-length. At the same time the spectrogram shows that this line has an extended structure* with at least two maxima. From figure 7 it is seen that the accuracy with which the wave-length can be determined is limited by the broad structure of the line itself; the precision of the scale of wave-lengths given by the sharp optical lines is too great to be fully utilized.

Further, it may be mentioned that the structure and wave-length are dependent on the chemical state; this has been shown by Glocker and Renninger† and was confirmed in these investigations. It is to be expected that such effects will be more pronounced in the still softer X-ray region accessible to the present methods. For the knowledge of the energy-levels in the solid state valuable information is to be expected from studies of this kind.

* During the course of these investigations similar results were found by Morand and Hautot: *Comptes Rendus*, **195**, 1383 (1932); **196**, 688 (1933).

† Renninger: *Z. f. Phys.* **78**, 510 (1932).

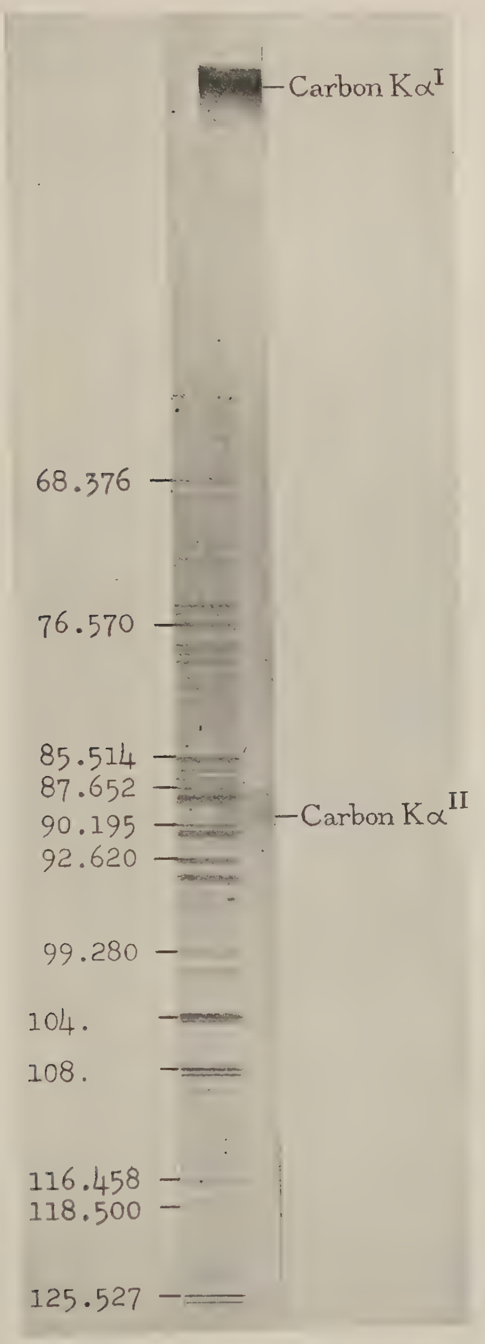
Carbon Kα^{I}
68.376
76.570
85.514
87.652
90.195
92.620
Carbon Kα^{II}
99.280
104.
108.
116.458
118.500
125.527

Figure 6.

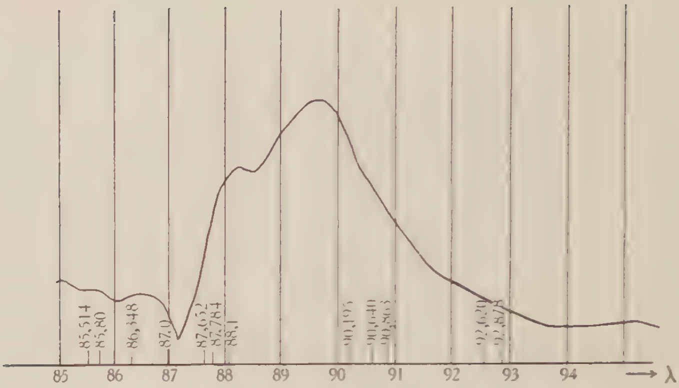

Figure 7. Photometrical registration of the Carbon $K\alpha_{II}$ line on the plate reproduced in figure 6.

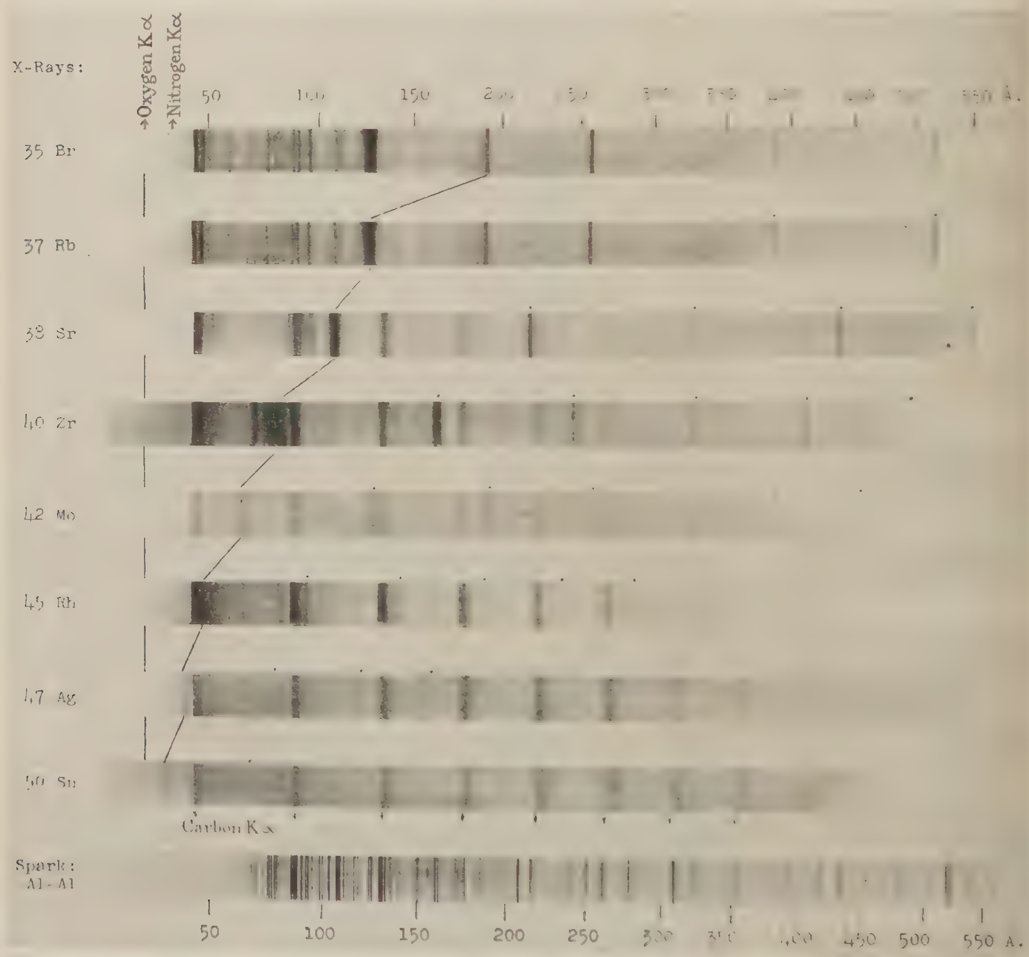

Figure 8. X-ray spectra from the elements 35 Br to 50 Sn in the very soft X-ray region 20–250 Å., and the optical spark spectrum of Al for comparison. Most of the plates show the Carbon K_α line in many orders.

§ 6. AN EXTENSION OF THE X-RAY SERIES

Of the three well-known X-ray series, the K series has been followed down to 4 (Be) at the wave-length 115 determined by Söderman in 1929 with the plane grating. The L series by the same method has been found down to 20 (Ca) at about 41 Å. by Kellström (1929). The M series in the same way was extended to 39 (Y) with one line at 93 Å. by Prins and Taken. Finally it may be mentioned that for the heaviest elements a few lines in this region (40 to 100 Å.) have been found which must be assigned to the N series or possibly to an O series.

During recent weeks some spectrograms of the elements 50 (Sn) to 34 (Se) were taken for a preliminary investigation. A set of these spectrograms is given in figure 8. In most of them the carbon K line (45 Å.) is to be seen in many orders; further, the oxygen line at 23·6 Å. and the nitrogen line at 31·6 Å. are more or less visible on all spectrograms.

A great number of hitherto unknown lines is to be found on the plates. The strongest line is due to the transition M_V–N_{III} (or M_{IV}–N_{II}) which for the four lowest elements 38 (Sr), 37 (Rb), 35 (Br) and 34 (Se) has the wave-lengths 109, 128, 193, and 230 Å., respectively, according to provisional measurements. For the lower elements the line is split up into two or more components. This same line has been measured, with a crystal-lattice for elements from 92 (U) to 63 (Eu) by Hjalmar (1923) and Lindberg (1931), and with a plane grating for the elements 51 (Sb) to 39 (Y) by Prins and Taken (1932).

§ 7. ABSORPTION SPECTRA IN THE VERY SOFT X-RAY REGION

The absorption spectra in the very soft X-ray region have only been studied in a very few cases. Thibaud (1928) registered by the plane-grating method the K absorption discontinuities for 8 (O), 7 (N) and 6 (C) at 23·5, 31·1 and 43·5 Å. respectively. With these exceptions the only measurements of soft absorption edges are those of Holweck (1927) with radiation which was not strictly monochromatic. The results of Holweck, though they are very interesting as a preliminary investigation in this field, are for methodological reasons not very accurate. Holweck's measurements include the K absorption limit of the elements 9 (F) to 5 (B) and the $L_{II,III}$ absorption of the elements 18 (A) to 13 (Al). For the last element at $L_{II,III}$, he found 68 volts, corresponding to 181 Å.

The concave-grating method as used in these investigations offers a very simple and at the same time very accurate means for studying X-ray absorption spectra in the soft X-ray region (above 50 Å.). Instead of using the continuous X-ray spectrum, which in this region is extremely feeble, the optical spark spectra from elements giving spectra rich in lines are used.

As an example of the applicability of this method a spectrogram taken by Mr Sanner, M.A., is reproduced in figure 9. As source a spark between copper electrodes was used. As is seen in the upper part, which was exposed without an absorbing screen, the line density is very high in this region, 150 to 200 Å. The lower part shows the effect of introducing an aluminium foil 0·5 μ. thick into the path of the radiation. This is practically completely cut off at 170·5 Å., which represents the L_{III}-absorption limit of Al. This value of the wave-length corresponds to $\nu/R = 5{\cdot}34$, which is in the very best agreement with the value $\nu/R = 5{\cdot}3$ computed from the X-ray emission spectrum of Al.

L_{III}-*absorption of* Al

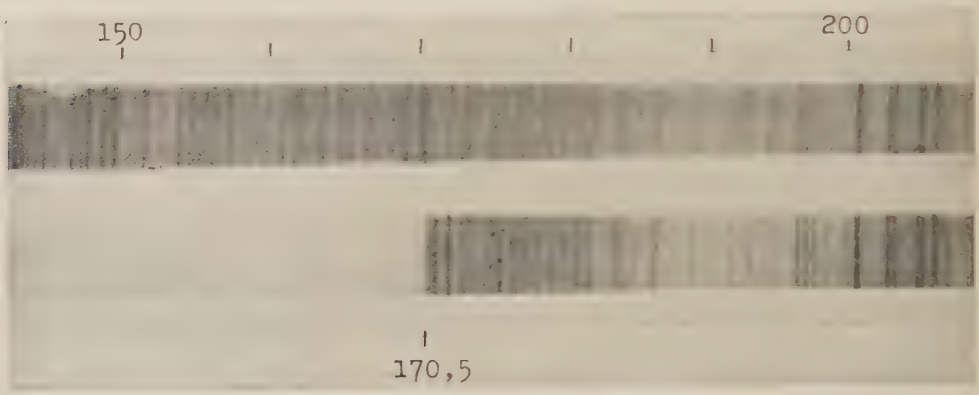

Figure 9. Spark spectrum of Cu-Cu, the lower part with Al-foil inserted. Time of exp. 10 min. and 50 min.

§ 8. ACKNOWLEDGMENT

I wish to thank Mr Torsten Magnusson, M.A., for valuable assistance during the experimental investigations here reported.

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STUDIES IN INTERFEROMETRY—II. THE CONSTRUCTION, TESTING AND USE OF REFLECTION ECHELONS FOR THE VISIBLE AND ULTRA-VIOLET REGIONS

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ABSTRACT. The practical details of the construction and methods of testing reflection echelons are considered. The various methods of mounting are compared. A method is described whereby the instrument can be used for wave-length measurements. It is further shown that the echelon provides an alternative means of standardizing length units in terms of a wave-length of light.

§ 1. INTRODUCTION

When Michelson⁽¹⁾ described the echelon principle thirty-five years ago, he specifically considered the possibility of using the grating as a reflection instrument. The fact that glass transmission echelons can only be employed for the visible and near infra-red regions would be a sufficient reason for using the reflection type. In recent years another real disadvantage of the transmission instruments has become apparent; when a resolving-power of the order of 10^6 is required, as in many hyperfine-structure problems, the transmission echelon becomes so bulky that it is difficult for the optician to mount the plates satisfactorily and for the user to maintain the requisite temperature equilibrium during an exposure.

Michelson foresaw one of the major difficulties in the construction of a reflection instrument, and remarked "Here the difficulty, even supposing the optical work to be perfect, would be the joining of the separate plates in such a way as to have always the same distance between them." Dr C. V. Burton attacked the problem in an entirely different way. Optically plane metal mirrors were separately arranged in echelon formation on a rigid base. Each plate was to be adjusted into its proper position relative to the previous one by means of finely controlled screw adjustments, the correct separation and parallelism being assured by electrical contacts. Dr Burton's untimely death during the War prevented the completion of the task. I was not able to obtain details of his work, and therefore experimented with three Fabry-Perot half-silvered plates. These were adjusted by means of Brewster interference fringes. I was forced to the conclusion that while it would be possible to make the adjustments to the requisite degree of accuracy, I could not think of any design of mount which could reasonably be expected to remain in permanent adjustment. Without this permanency, the large number of plates would require considerable time to be spent in adjusting, and a record would not be trustworthy unless after the exposure the adjustments proved to have been correct. It was with great reluctance that this method was abandoned, since it would be very much less costly than any other, and the plates, once made, could be used again in a different mount to obtain an echelon of different plate-separation.

It has been long known in the optical industry that when two optically plane glass plates are carefully cleaned and freed from dust they will adhere when pressed together. This type of optical contact was first referred to by Twyman(2), and it forms the basis of the optical adhesion described by Parker and Dalladay(3), in which the contacted plates are lightly clamped and are heated in an electric oven to a predetermined temperature. Apart from the heating, the contact is so intimate that no light is reflected at the interface when the plates have the same refractive index. From the measurements of Johnnot(4) on the thickness of the black spot in soap films, it follows that the surfaces are separated by an exceedingly small fraction of a wave-length.

All transmission echelons made in this country since 1908 have been contacted in this way in order to minimize loss of light by reflection at the interfaces. For transmission, the optical criterion is that $(\mu - 1)\, t$ should be constant for each plate, so that small variations in the refractive index μ have to be compensated for by suitable variations in the plate-thickness t. The plate-surfaces are therefore not uniformly plane, so that the optical contact cannot extend over the whole interface. It is however sufficiently general to ensure that the loss of light by interface reflections is small and that the superposed "Fabry-Perot" fringes observed by Stansfield(5) and Kent and Taylor(6) are vanishingly weak.

μ
t

For a period I considered that the sole reason why transmission echelons could not be used as reflection instruments was that these small variations occurred in t, and that if an instrument with plates of constant metrical thickness were to be constructed it would give good definition. Very occasionally, in particularly good meltings of optical glass, the refractive index is so uniform that practically no "touching up" or local polishing of the large plate is necessary. During 1927 a glass echelon of this kind became available. It gave particularly good definition as a transmission instrument, but when it was used as a reflection instrument the results were very poor. Allowance had of course to be made for the fact that the resolving-power and the dispersion were about three to four times greater than in the case of transmission.

The explanation of the discrepancy became obvious when the echelon was tested with the Twyman-and-Green interferometer as described below. The contacting of two plates under pressure causes a slight distortion, which becomes more and more pronounced as successive plates are added. The geometrical echelon formation that is essential seems to make this curvature effect unavoidable since the optical contact has to be obtained by pressure. This curvature has no appreciable effect on the performance of the echelon for transmission work, but makes it practically useless for reflection work. In general, contacted-glass echelons give astigmatic images by reflection.

A solution of this difficulty came accidentally from another investigation in which a Fabry-Perot etalon was crossed with a spectroheliograph(7). Quartz and fused-silica plates were employed instead of glass in order to avoid any distortion due to the heating effect of the comparatively strong beam from the collimator of the spectroheliograph. As it was found extremely difficult to adjust the parallelism of the etalon plates in position on the instrument, I decided to try to contact three plane parallel separators cut from the same strip of fused silica. To my surprise these contacted easily on either the quartz or the fused-silica plate without any pressure being necessary, and the interference rings observed in the neighbourhood of the distance pieces showed no distortional effect. This type of contact in quartz or silica is of a very different nature from that obtained with glass. A glass contact can generally be separated by a sharp blow or gentle pulling of the parts. In this instance it was only with considerable difficulty that the etalon plates could be forced apart; the small separators, measuring 4 × 4 × 3 mm., could only be removed by soaking the plates in petrol for some days, after which they were easily detached.

It was first thought that this improved type of optical contact was entirely due to a greater degree of planeness of the surfaces to be contacted. When a plane surface is being polished the frictional heat makes it convex, and in a great measure the art of the optician lies in working his surface to the requisite degree of convexity so that it will become plane when the temperature is allowed to become uniform throughout the material. This effect is much smaller in quartz and fused silica than in glass for two entirely different reasons. In quartz, the thermal conductivity is approximately twenty times greater than in optical glass, so that the temperature-gradient from the polishing surface is correspondingly reduced; with silica, the almost negligible expansion-coefficient makes the larger temperature-gradient of little importance. Plane surfaces are much more easily obtained with quartz or silica than with glass.

After repeated trials a pair of glass plates which had an even greater degree of planeness than the above-mentioned plates was obtained. Although the glass plates could be cleaned with dilute acid more thoroughly than with quartz or silica plates, an optical contact could not be obtained in this case without pressure. In order to avoid temperature distortion in the handling of the plates asbestos gloves were worn. There seems to be some fundamental difference, for which no explanation can at present be offered, between the optical contacting of glass on one hand and of quartz or silica on the other.

This distortionless contacting of quartz or silica indicated that if plane parallel plates of equal metrical thickness could be constructed, a reflection echelon would become a practical possibility. A rough trial made with a few plates showed that the successive contacting with silica did not introduce any further practical difficulties. Through the co-operation and enterprise of Prof. Takamine in venturing to order an instrument built on these lines from Messrs A. Hilger, it became possible to make a full-scale test of the practicability of the method.

§ 2. CONSTRUCTION

Experience had shown that as far as the avoidance of distortion was concerned, either quartz or fused silica could be employed. With glass echelons the practice has been to work a single large plate as plane as possible, provided it has a uniform retardation $(\mu - 1)\,t$ over all its parts. This plate is then cut into the smaller pieces to form the complete echelon. Single plates of sufficient area cannot be obtained in quartz, and as silica in large plates is never perfectly annealed, small distortions inevitably occur when a larger plate is cut. In both cases therefore the individual plates have to be made singly.

Provided one could be certain that the refractive index was sufficiently uniform, the constancy either of $2(\mu - 1)\,t$ or of $2\mu t$, determined respectively by the Twyman-and-Green interferometer and the Fizeau interferometer*, would suffice to yield the requisite constancy of $2t$ needed for the reflection echelon. It is well known that quartz crystals from different localities have slightly different indices, and it could not be safely assumed without further experiment that the index is sufficiently uniform over the different parts of a single large crystal.

One of the principal difficulties in obtaining long-exposure photographs with transmission echelons is that of keeping the temperature of the echelon enclosure within sufficiently close limits. This difficulty obtains in spite of the fact that the positive expansion of the glass is to some extent compensated for by the temperature variation of the index, which is negative both for glass and air. In most instances some form of thermostatic control must be employed. Other things being equal, the higher the resolving-power of an echelon the closer are the limits within which the temperature has to be maintained. It is by now fairly generally realized that a prismatic spectrograph will never show the same standard of definition, and consequently of resolving-power, when it is used photographically as when it is used visually. This is the case with a spectrograph having a camera lens of sufficient focal length to ensure that the grain-size of the photographic plate shall not affect the definition. The generally accepted explanation is that the prism is subject to minute temperature waves in the material; the resulting small oscillations of the spectral line are recorded as a broadening of the photographic image, and this cumulative effect is not detected in visual observation. This reduction of resolving power in prismatic spectra arises from the variation of the refractive index with temperature, and correspondingly it would occur in a reflection echelon as a result of variation of the plate-thickness with temperature. Hence it is obvious that material having the lowest possible coefficient of expansion should be used, provided it will give the distortionless optical contact that is essential. Invar steel apparently will not contact in this way, so that the plates must be made of fused silica.

The standard of accuracy of workmanship in the reflection instrument is between three and four times higher than that in the transmission instrument when used in the same spectral region. The principal field for the use of the reflection form is the ultra-violet and possibly in the Schumann region, which is as yet unexplored so far as high resolving-power is concerned. A standard at least ten times higher than before has to be aimed at; this implies the taxing of the optician's skill to the utmost, and the development of special methods of testing which would show up the minute errors that are no longer allowable. It was therefore considered desirable that the plates should all have the same area; such a pile would have greater rigidity than the usual wedge-shaped pile, and the increased cost of the material would be more than compensated for by the greater ease and convenience in working and testing the plates. The minimum plate-thickness should not be less than about 4 mm.; thinner plates are liable to distortion in working unless the area of each plate, at present 43 × 36 mm., be reduced. The maximum plate-thickness is primarily governed by the total height of the complete echelon; to ensure the necessary rigidity, the cross-sectional area of each plate should be increased if the total height of the pile exceeds 25 cm.

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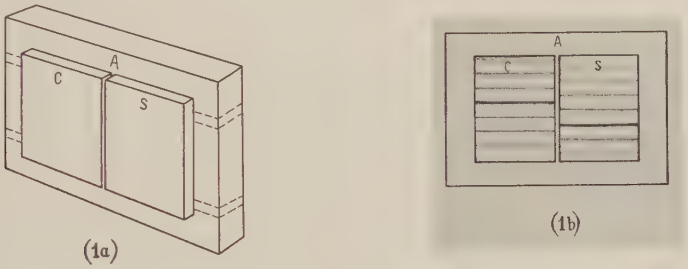
* It will be observed that a determination of the retardations $2\mu t$ and $2(\mu - 1)\,t$ for a plate will give both μ and t implicitly. This method has long been in regular use by Messrs Hilger to determine quantitatively the local variations in the index of a melt of optical glass.



(1a) (1b)

Figure 1.

The plate-thickness of the first instrument was chosen to be slightly greater than 7 mm. This would give one of the main components of H_α, midway between the orders of the other component, so that the separation between these components could be measured without any error due to the Oldenburg *Schrumpfungs Effekt*. This important problem still remains to be finally solved by the employment of adequate resolving-power together with suitable cooling of the light source with liquid hydrogen or helium instead of liquid air.

The silica plates are first roughly cut and ground to the required size and thickness. Originally the long edges were left grey or matt, but as it was found that the matt surface caused appreciable scattering, especially in the far ultra-violet, the long edges in some of the later instruments were optically polished; as a further precaution, in future these edges will be optically worked so that they make angles slightly less than 90° with the working face of the echelon plate. After the polishing process the inevitable differences in thickness between the various plates were determined in the following way.

Two plates S and C, figure 1 (*a*), were placed in optical contact side by side on a larger plate A of quartz or silica, the surface of which was known to be plane. The composite mirror was then set up as one arm of a Twyman-and-Green interferometer, in place of *E*, figure 2, and the appearance of the field of view was in general as shown in figure 1 (*b*), where the central fringe for *C* is displaced approximately 1·7 fringes from that of the standard *S*. A slight downward pressure on the arm carrying the unit causes the optical path in that arm to increase, and if in consequence the fringes also move downwards the plate *C* is approximately $0{\cdot}8\lambda$ thicker than *S*. A more accurate value can be obtained by substituting monochromatic radiation for the white light of the interferometer. The different plates were thus measured in terms of the standard *S*. Finally, the thinnest plate is chosen as the standard and all the other plates were worked down to this thickness by repeated polishing and testing. If by accident a particular plate be polished so that it is thinner than the new standard, either that plate must not be used, or all the other plates must be reworked down to its standard.

The work of contacting, removal for reworking, and recontacting the plate on *A* is extremely laborious, and attempts were made to reduce the area of contact by employing plane strips or rails, indicated by dotted lines in figure 1 (*a*), instead of a plate. It was found that the thickness of the plates worked by this control steadily increased; each optical contact involves a slight wear of the narrow rails, and in time the wear becomes large enough to be measurable. Messrs Hilger have developed a new type of double interferometer which obviates the necessity for continual recontacting of the plates*. This arrangement has been described by them in detail elsewhere. The writer is at present investigating the possibilities of a modified form of this interferometer which should enable a still higher degree of accuracy to be attained. This will be described in a subsequent paper in this series.

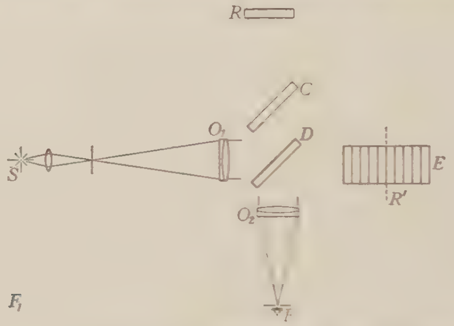


Figure 2. Twyman and Green interferometer test of reflection echelon.

§ 3. MOUNTING

When it is realized that a good optical contact of the type under consideration is nearly as strong and rigid as the material itself, one would not expect appreciable distortion in a pile of the cross-sectional area employed when the length is of the order of 20 cm. Yet if the echelon is supported at its ends with the steps horizontal, the sagging of the centre manifests itself in a considerable change of focus of the diffractional pattern. In accordance with a suggestion made by Mr Twyman the pile rests on narrow fibre strips inside an optically worked rectangular box of invar steel. In this way each plate receives its share of support, and the strip is so thin that no perceptible movement of the echelon in its box can be detected.

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* British Patent Specification, No. 367859 (Twyman and Dowell).

§4. THE TESTING OF REFLECTION ECHELONS

The performance of a line grating, a Lummer plate, or even a transmission echelon can be immediately tested by examining the definition of the patterns obtained for spectral lines from selected sources. One knows from long experience what standard should be obtained with the green mercury radiation if the workmanship be sensibly perfect. The resolving-power of a large reflection echelon is, however, so high that this direct test is much more a test of the pressure, temperature, and current-density in the mercury lamp than of the instrument. It is quite probable that a systematic search would lead to a critical resolving-power test, in the same way that the clean resolution of the Fe triplet at λ 2749 Å. is at the present time regarded as a critical test for a large back-reflecting quartz spectrograph (E 1).

A test of this nature, if not satisfactorily passed, yields little information as to the nature of the defect and the position of the defective plates, so that a more direct method is highly desirable if not essential. The first method adopted is shown in figure 2. The echelon E was mounted as one mirror arm of a Twyman-and-Green interferometer and the green radiation from the mercury source S was selected by placing a suitable filter at F, the focal plane of the second objective O_2. The successive steps (1 mm. at a distance of about 100 cm.) were rather difficult to see clearly, so that either a magnifying field lens (not shown in the diagram) was used, or a special short-focus lens was substituted for the standard 60-cm. objective O_2 and the echelon was mounted as close to the dividing mirror as possible. The reference mirror R was adjusted so that its image in D coincided with one of the step faces of the echelon.

When the image R' makes a small angle with the echelon step in the horizontal plane, the pattern observed in the field of view is that shown diagrammatically in figure 3 (*a*). Since $2t$ will not in general be an exact integral multiple of the mean wave-length of the central part of the green line, the fringes from successive steps will not be coincident.

A wide variation in the clearness or the visibility of the fringes from the different steps is immediately apparent. If the image of the reference mirror practically coincides with, say, the twelfth step, the fringes here are most distinct, while those of the eleventh and thirteenth steps are only slightly less clear. The falling off in the visibility of the tenth and fourteenth-step fringes is more marked, and after the eighth and sixteenth steps the fringes are too ill defined to be seen with certainty.

By suitably tilting the reference mirror R the now inclined fringes can be made to run into each other as shown in figure 3 (*b*). The resultant composite fringe is however, not straight, but has the S formation shown in figure 3 (*c*). The immediate deduction was that the plates were distorted; that while plates 11, 12 and

13 were sufficiently uniform, 8 and 15, and still more 9 and 16, had different values of effective plate-thickness. This interpretation was proved to be incorrect by displacing the reference mirror so that its image coincided with the ninth step. While the eighth, ninth and tenth-step fringes were now in line, the fringes in the eleventh, twelfth and thirteenth steps were curved to the right as in the upper part of the ʃ sign, but the fringes in the same three steps were curved in the opposite direction when the reference mirror was moved so that its image coincided with the fifteenth step. Thus this curvature was independent of the echelon and arose simply because of the asymmetry of the intensity-distribution in the green-line source. The red radiation from a Michelson cadmium lamp was then tried as a source. This is much less bright than the mercury light, and in this type of interferometer, where the light from a pinhole has to be spread over the whole field of view, the fringes were extremely faint, but as far as one could tell the composite fringes appeared to be straight.

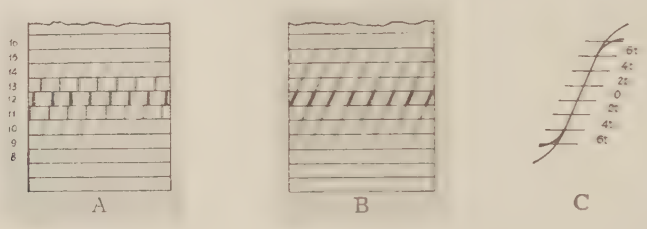

Figure 3.

It will be realized that a photograph of these composite fringes from a perfect interferometer and echelon would suffice to give the visibility and phase curves required to determine uniquely the distribution of an asymmetric spectral line[*]. The visibility data could be obtained from microphotometer curves of the fringes in the various steps and the phase curve by measuring the deviation of the outer fringes of the outer steps from the alignment set by the central-step fringes. As a method this is only of academic interest, since the available resolving-power would only be that of the echelon itself, which will give objectively all the information that would have to be deduced by a Fourier analysis from the above-mentioned data.

The above experiment has been described in detail since it is almost identical in principle with that of a method devised by Kösters(8) for standardizing the metre. In his method a ┌─┘ -shaped gauge is substituted for the echelon E in a Twyman-and-Green interferometer as in figure 2. The reference mirror image is placed midway, so that the arrangement is equivalent to bringing steps 8 and 16 of figure 3 (a) into juxtaposition. It will be obvious that the phase displacement of the fringes due to any asymmetry of the line source will be doubled and can introduce a systematic error far in excess of the possible errors of observation. These phase displacements have long been known and studied by metrologists, but the simple interferometer, whether of the Michelson or of the Twyman-and-Green type, only enables the *difference* between the phase curves of any two sources to be determined. It appears to the writer that results obtained by this method cannot be relied upon unless also the spectral line employed can be shown to be symmetrical to the required degree of accuracy. The same criticism will apply to some extent to wavelength measurements with Fabry-Perot interferometers with lightly silvered or platinized surfaces, in which the poor reflecting coefficient of the films is apparently compensated by the use of a high order of interference. A fairly close analogy would be afforded by the problem of finding the position of a variable binary star by means of a telescope. With a small objective aperture the diffractional image would appear symmetrical, with the centre of light-intensity of the binary as the axis. As the aperture is increased the pattern becomes more and more asymmetric, but the true positions of the components cannot be found until they are cleanly resolved, since they may differ both in magnitude and in intrinsic brightness.

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[*] Michelson, *Studies in Optics*, pp. 39, 40.

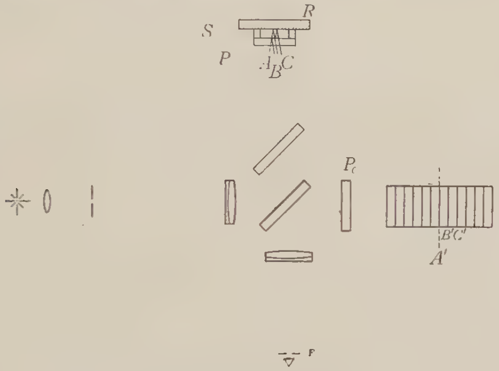

Figure 4. Combination of Fabry-Perot and reflecting echelon in Twyman-and-Green interferometer.

The following method, shown in figure 4, was devised to eliminate the possibility of spurious results arising from asymmetry of the source; it functions almost equally well with a white light source. The silver was removed from all but the centre portion of the plane quartz reference mirror R, and the two halves of a spare echelon plate S were contacted to it. A plane parallel quartz plate P was lightly silvered and contacted to the silica separators, the silver being carefully removed from the parts of P to be contacted to S. The unit then formed an interferometer similar to that of Benoit, the precursor of the arrangement of Fabry and Perot, but in the latter the fringes are observed by transmission instead of by reflection, as in the original Benoit method. This unit was mounted in place of the movable mirror in the Twyman and Green interferometer, and a compensating plate P_c of the same optical thickness as P was added to the echelon arm as shown.

The unit was adjusted until the image of the silvered surface R coincided with the plane of one of the steps A'. The usual white-light fringes could be seen in that step; a similar set in the next step B' was given by the light which had undergone one internal reflection in the mirror unit, while the fringes in C' corresponded to two such reflections. If the optical-path difference of the unit (allowing for the effect of phase changes at reflection) is equal to twice the plate-thickness of the echelon, the fringes will be in alignment and the appearance of the field of view will be that shown in figure 5. The separators were cut from a slightly thinner and therefore useless echelon plate. By chance this happened to compensate almost exactly for the phase-change at the inner surface of *P*, so that the fringes from the consecutive steps were in line. The fringes rapidly decrease in clearness, so that at the fifth step only the central (zero-order) fringe can be seen. This decrease is due in this case to the fact that while the amplitude of the wave reflected at the various echelon steps is constant, that reflected from the unit decreases rapidly with each multiple reflection. Any lateral displacement of the fringe system at any one step would show that step to be either too thick or too thin. A variation in the separation of the fringes would mean that the plate was not parallel. By tilting the movable unit, the fringes could be arranged so that only a single horizontal zero-order fringe appeared in each step. Provided the step faces are strictly parallel in the horizontal and vertical planes, these fringes should be parallel and occupy exactly corresponding positions in each step face. The echelon was tested every five steps in this way, and after the final assembly no detectable displacement or lack of parallelism could be found in any of the steps. This is not merely a test of the metrical equality of the plates but of the real optical requirement that the surfaces of the consecutive steps have equal separation. If the equality and parallelism of the separate plates are definitely assured from previous tests with the double interferometer, this becomes a test of whether the optical contacting of the plates has been uniformly successful.

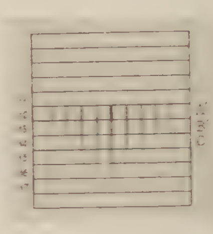

Figure 5.

Neither of these tests provides a means of guarding against another possible source of error—a uniform variation in the plate-thickness which, while imperceptible in two or three successive steps, might amount to an appreciable fraction of a wave-length in a complete echelon of 30 to 40 plates. If the separators of the etalon unit were made five times thicker than the echelon plates, white light fringes would be visible at the first, fifth, tenth, etc. plates, and this result would prove the absence of any uniform variation. Fortunately this was not necessary since the primary effect of a uniform variation in the optical-path difference is to cause a change of focus of the image. This can become comparatively large before the standard of definition is appreciably reduced. This point has been discussed by Twyman(2) in connexion with the effects of clamping pyramidal transmission echelons and is further exemplified by Koláček(9) in an analysis of slightly wedged-shaped Lummer plates. Some difficulty due to this effect was at first experienced when the echelon was tested in its mount. Instead of coming to a focus at the focal plane of a 170 cm. objective, the pattern was focused about 5 mm. beyond; the definition appeared as good, however, as when the echelon was tested without its mount, in which case the pattern was correctly in focus at the focal plane of the lens. An alteration in the design of the mount enabled this error to be reduced to a fraction of a millimetre.

§ 5. EFFECT OF IRREGULARITIES IN THE STEP-WIDTH

For an echelon of step-width s and plate-thickness t, the separation between consecutive edges k is $(s^2+t^2)^{\frac{1}{2}}$. If α and β are the angles of incidence and diffraction as measured from the normal to the diffracting surfaces, the angles of incidence and diffraction measured from the normal to the plane containing the edges of the echelon steps are $(\gamma+\alpha)$ and $(\gamma+\beta)$. From the simple theory of the line grating, the path difference between beams from consecutive steps is:

s

k, α, β

$$k\{\sin(\gamma+\beta)+\sin(\gamma+\alpha)\} \qquad \ldots\ldots(1).$$

Since $\sin\gamma = t/k$, and $\cos\gamma = s/k$, the equation for reinforcement becomes:

$$t(\cos\alpha+\cos\beta)+s(\sin\alpha+\sin\beta)=m\lambda \qquad \ldots\ldots(1a).$$

m

The incident and the diffracted beams invariably make very small angles α and β with the reflecting step face, so that the expression, without any appreciable loss of accuracy can be further simplified into

$$2t+s(\alpha+\beta)=m\lambda \qquad \ldots\ldots(2).$$

s should be constant as well as t, since a change δs in the step-width will cause a variation $\delta\beta$ in the angle of the diffractional maximum given by

$$\frac{\delta s}{s}=-\frac{\delta\beta}{(\alpha+\beta)}.$$

If a spectral line is in single-order position $\alpha=-\beta$, and no loss of definition due to any variation of s, however large, can occur. In double-order position $(\alpha+\beta)=\lambda/2s$, since λ/s is the angle between the orders, while for the outer orders, if visible in this position, $(\alpha+\beta)=3\lambda/2s$; the tolerances that are allowable in s to secure sensibly perfect definition of the central orders must be reduced threefold if the same standard is to be maintained for the outer orders.

That small variations of the step-width do exist can be easily seen by direct inspection, and the effect is even more prominent in glass transmission instruments where the contacting is more difficult to achieve. The effect was studied in some detail with the third and fourth instruments, using a water-cooled quartz mercury lamp (Hansen type) as a source. In neither case could any difference be detected in the standard of definition of a component when it was changed from single to double-order position. On the other hand, the definition of the outer orders was invariably inferior to that of the central orders. It would be a great convenience in rapid working if these orders could be utilized. The centre of gravity of a somewhat broad line or an unresolved group is spuriously displaced by the instrument when the line is in double-order position. This is due to the fact that the amplitude curve for a single slit varies rapidly at these positions, while those of the outer orders nearly coincide with the positions of the secondary maxima and are not perturbed by this effect. Suppose, for example, that the pattern consists of one or two weak lines placed approximately midway between the orders of a much stronger group. The weak lines could be examined in single-order position, and while the double-order image of the strong group may be much over-exposed, the outer orders of this group would be at a suitable intensity for measurement. The writer

intends to try the experiment of measuring the step-widths of an echelon with a travelling microscope or cathetometer and to mount ┌─┘-shaped diaphragms on the sides of the steps. These diaphragms can be machined to the requisite dimensions
a for each individual step, so that all the apertures have both uniform width a, and
s uniform lateral spacing s. It is only when the gap or obstructing part between any two consecutive steps is small compared with the width of a reflecting face that the instrument behaves as a true echelon. When a line is in double-order position, the angles that the various orders make with the normal to the step face are $\pm 0{\cdot}5\lambda/s$ and $\pm 1{\cdot}5\lambda/s$. The maxima for a single slit occur at 0 and $\pm 1{\cdot}43\lambda/a$. These values will also be correct when the light is incident on the grating at a small angle α, provided the direction $-\alpha$ is taken as the axis for measurement.

If the obstruction be chosen so that $1{\cdot}43s = 1{\cdot}5a$, the outer orders of a line in double-order position will coincide with the point of inflection of the amplitude curve of a single slit, and a somewhat broad line in this position will not be spuriously displaced.

In an experiment to test the practical importance of this point a high-pressure neon tube was used as a source, so that all the lines were somewhat broad. It was found that the distances between the inner orders of lines in double-order positions were approximately 1 per cent less than a third of the corresponding distances between the outer orders. In a similar test with a sharp satellite of the green Hg line no difference could be observed.

§ 6. CONSTANT-PRESSURE CHAMBER

It has long been known that extended exposures with the Fabry-Perot interferometer, for example, are frequently unsuccessful because of variations in the pressure of the atmosphere during the exposure. If an accuracy of 0·001 Å. is required for a wave-length 5000 Å., the variation of the atmospheric pressure during an exposure must not be such as to produce a variation of 1 part in 5×10^6 in the refractive index of the air. This corresponds approximately to a pressure-difference of 0·5 mm. of mercury, a difference that occurs only too frequently in a long exposure over a few hours. Even with an ordinary line grating this effect is a source of trouble and Merton(10) has devised an ingenious scheme whereby the effects of temperature-variations and pressure-variations can be cancelled during the exposure by means of a relay which introduces or removes an auxiliary heating system to compensate automatically for them. When still higher accuracy is required, the pressure-tolerances are correspondingly reduced; thus Perard(11) has recently given the mean wave-length of one of the krypton lines to the fifth decimal place in Ångstrom units. For this to have an absolute physical meaning, the pressure should be constant and specified to within 0·005 mm. The permanency associated with such optically contacted etalons and reflection echelons would enable these instruments to be placed *in vacuo* and this would also reduce any temperature disturbances. This idea was incorporated in a patent specification(12) filed on June 11, 1928.

In the initial tests of the first instrument an apparent disadvantage of the reflection form was encountered. The change from single to double-order position for the transmission type can be easily effected by a small tilt of the instrument so that the optical retardation of a plate can be increased by half a wave-length without alteration of the direction of the transmitted beam. In the reflection instrument only half this tilt is required, but the reflected beam is now deviated out of the field of view. This point could hardly have been foreseen, since no measurement of the tilt angle is normally made with the transmission instrument. Mr Twyman* at once proposed, in a letter dated October 26, 1928, to vary the pressure in order to change the order positions. He stated that this plan was suggested by consideration of the vacuum etalon, and he calculated the pressure differences required. He pointed out in his letter that since the density of the air would remain unaltered, its refractive index would not change and a small variation of temperature would be immaterial owing to the smallness of the expansion coefficient of silica. The method when tried proved very successful, and with it, extended exposures could be relied on to give as good definition for the weaker lines as were given by the short exposures for the stronger sources.

§ 7. METHODS OF USING REFLECTION ECHELONS

The obvious way to combine a reflection echelon with the necessary auxiliary dispersion system is to use the echelon as the mirror in a back-reflecting, Littrow type of spectrograph. Thus, if the reflecting film on the back of the quartz prism of the Hilger E 1 spectrograph be removed the echelon can be mounted behind the prism, and such an arrangement would be most compact and economical of light. This method was originally contemplated during the construction of the first instrument, but during the testing the disadvantages of the arrangement became apparent.

When an autocollimating telescope with a horizontal slit is used to observe the diffraction fringes from the echelon, it will be seen that while the fringes are horizontal at the end nearer the slit, towards the edge of the field the fringe curves downwards or upwards, according to whether the echelon stair ascends or descends away from the objective. If θ is the angle which the incident beam makes with the normal to the step faces of the echelon in the horizontal plane (i.e. with a line parallel to the long edges of the step apertures) equation (2) has to be modified into:

$$2t\cos\theta + s(\alpha+\beta) = m\lambda \qquad \ldots\ldots(3),$$

which may be rewritten

$$2t(1-\tfrac{1}{2}\theta^2) + s(\alpha+\beta) = m\lambda.$$

For any given fringe, since the incidence angle α is constant,

$$(t\theta^2/s - \beta),$$

should be constant.

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* This detailed explanation is given because D. A. Jackson, *Proc. R.S.* A. **128**, 508 (1930), has described the echelon and pressure chamber without any acknowledgment other than that the echelon was made by Hilger's. The complete patent specification was filed on March 8, 1929 and accepted on May 30, 1929.

The slope $\partial\beta/\partial\theta$ of the fringe

$$2t\theta/s \qquad \ldots\ldots(4).$$

This simply means that if the light is incident obliquely on the echelon edges, the reduced path difference is compensated for by a larger diffraction angle as measured from the normal of the plane containing the edges of the steps. The same effect should occur with ordinary line gratings, but much larger skew angles would have to be chosen before the effect became apparent. In the echelon the ratio t/s is about 7 while in the line grating it is of the order of unity.

Only in a few cases would it be practical to have the dispersions of the spectrograph and the echelon parallel, so that in general the dispersions must be crossed. In the usual quartz spectrograph such as the Hilger E 1 the slit is effectively at the side of the plate. If a reflection echelon be used as a mirror behind the prism, the light falling on it must make increasing angles in the horizontal plane as the wavelength decreases. With a horizontal cross slit, the echelon fringes become more and more inclined, in accordance with equation (4). It can be easily shown that as the slope becomes pronounced the available resolving-power is correspondingly reduced.

Suppose we consider a point P on the narrow horizontal slit, illuminated by monochromatic light, the central diffractional image seen with the echelon steps horizontal will be a rectangle $ABCD$, figure 6, the angular width of AC in the direction of the dispersion being $2\lambda/Ns$ while in the horizontal direction AB has an angular width $2\lambda/h$, where h is the length of the step. Another point Q on the slit will have as its image the rectangle $EFGH$, which is displaced upwards through the angle $\delta\beta$ on account of the variation in θ. Other points, between P and Q on the slit, will have their rectangular diffraction patterns between these rectangles. The resultant fringe is therefore contained between AH and DF, and the angular width of the fringe in the direction β of the dispersion is proportional to DH instead of to AC. With a slope $\partial\beta/\partial\theta$ the angular width of the fringe is increased in the ratio HD/AC, and the resolving-power is reduced in the inverse ratio. It will be seen that

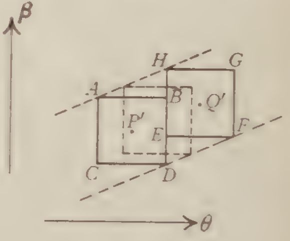

Figure 6.

$$\frac{\text{resolving-power when } \theta = \theta}{\text{resolving-power when } \theta = 0} = \frac{AC}{HD} = \frac{1}{1 + (2Ns/h)(\partial\beta/\partial\theta)} \qquad \ldots\ldots(5).$$

If we assume that we are prepared to accept a 10 per cent reduction of the resolving-power, the maximum permissible value of $\partial\beta/\partial\theta$ according to equation (5) becomes $h/18Ns$. Substituting this value in equation 4, the maximum value of θ should not exceed $\bar{\theta}$ where

$$\bar{\theta} = h/36Nt \qquad \ldots\ldots(6).$$

This being the angle of incidence on the horizontal plane, the beam falling on the photographic plate will make an angle twice as great with the incident beam, so that the total angular range θ over which the resolving-power is not reduced by more than 10 per cent is $h/18Nt$. For reflection echelons of plate-thickness 0·73 cm., and height 4·0 cm., this angle only amounts to 40′ in a 25-plate echelon and 25′ with a 40-plate instrument.

The width of the aperture perpendicular to the axis of the quartz lens of the standard E 1 spectrograph varies from about 8 cm. to 12 cm. according to whether the spectrograph is set for the first (visible) position or the fourth position for the lowest wave-length region. If the lens had been achromatic so that the plate is normal to the axis, only the patterns in the first 2 cm. or 1·3 cm. of the plate nearest the slit would have the required standard for a 25-plate or a 40-plate echelon. Table 1 gives the approximate lengths of the 25 cm. plates over which the resolving-power should be 90 per cent or higher.

Table 1

Focal length of objective (cm.)	Number of plates in echelon	Useful length of plate	
		First position visible (cm.)	Fourth position (cm.)
170	25	6·0	4·0
	40	4·3	2·9
300	25	10·6	—
	40	6·5	—

The complete range of the E 1 instrument is usually obtained in four settings; with this method of using the reflection echelon at least 20 positions would have to be determined with a 25-plate echelon, and correspondingly more for more powerful instruments. If a 300-cm. objective be used in place of the 170-cm. lens with a 40-plate echelon, about 50 positions would have to be standardized with reference to lens, prism-position and plate-tilt.

If the spectrograph were redesigned so that the slit lies either above or below the axis of the lens, the above ranges would be automatically doubled. In the above calculations it has been assumed that the slit is in effect at the exact edge of the plate. Since the totally reflecting prism is a few centimetres distant from the slit, allowance has to be made for the spread of the beam so that the extreme edge of the plate does not correspond to the zero angle of incidence on the echelon, as it has been assumed to do for the purposes of the above table.

A further defect of this arrangement, that has not been fully investigated, arises from the fact that the slit is placed to give a parallel beam on the prism for the wave-lengths corresponding to the centre of the plate. For wave-lengths at each end of the plate, the slit is approximately ± 10 cm. beyond the focal plane, and the beam incident on the echelon is not parallel. With a 170-cm. objective the planes of the images focused by the extreme steps of a 40-plate echelon differ by ± 1·3 mm. from that reflected by the central step. This is much more than the experimental depth of focus with a 4-cm. aperture so that the light from the outer steps is effectively spread over a larger horizontal strip and these steps do not contribute their full quota for interference purposes. The resolving-power is thereby lessened in the same way as in the case of the Lummer plate. The main disadvantage of this effect is that it is at a maximum at the end of the plate, which, according to the above table, is the only part that can be employed to give approximately full resolving-power. If by means of an extension tube the slit were moved 7 cm. outwards and the lens were suitably displaced, the beam incident on the echelon would be sufficiently parallel for that portion of the plate which can be usefully employed.

A further disadvantage of this type of mounting is that it cannot be employed for standard wave-length measurements. This, in the writer's opinion, is probably the most important function of the reflection echelon, since for general hyperfine-structure work two or three such echelons of different plate-thickness should be available, so that the effect of overlapping orders can be avoided.

For these reasons, this method was abandoned and the principle of projecting the echelon pattern on the slit of the spectrograph adopted. This involves considerable loss of light in the extra reflection and refraction surfaces involved, and increased difficulties in coupling two separate instruments, but the advantages far outweigh the disadvantages.

Either a lens or a concave mirror can be employed to project a parallel beam on the echelon and to bring the diffracted beam into focus on the slit of the spectrograph. The mirror method is free from all difficulties of chromatic aberration, and although it does not permit as adaptable a mounting as the lens it would be much easier to construct; the reflecting-power of platinum in the ultra-violet is only about 33 per cent, and since two such reflections have to occur the final image will have an intensity of approximately $\frac{1}{9}$ of that possible when a lens is employed instead of a mirror. For that reason alone, in view of the necessity for collecting all the light possible, the simpler mirror system was abandoned for the more difficult lens method*.

Figure 7 shows a diagrammatic side view of the arrangement. The light from the horizontal slit S_1 is totally reflected by the prism P and emerges from the lens L as a parallel beam. The light diffracted from the echelon E is in turn focused across the vertical slit S_2 of the spectrograph, which makes a coarse analysis in the horizontal plane. With a properly constructed echelon the definition is equally good over the whole of the plate provided that the lens L is sufficiently achromatic. A quartz-fluorspar doublet was originally used but proved insufficiently achromatic, and a quadruple lens of quartz-fluorspar-sylvine-quartz has been designed by Mr Perry(14) of Messrs Hilger. In this objective the thickness of each component has been chosen with the object of reducing the chromatic aberration, the values of which are shown in curve b of figure 8. Curve a is that of the quartz fluorspar doublet. It might be remarked here that the construction of such an objective

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* Since this was considered, Hocheim has developed a method of obtaining a high-reflection-coefficient mirror in this region and Ritschl(13) has employed a Hocheim mirror instead of a lens. The reflection coefficient falls off very abruptly at 2300 A., long before the limit for quartz is reached.

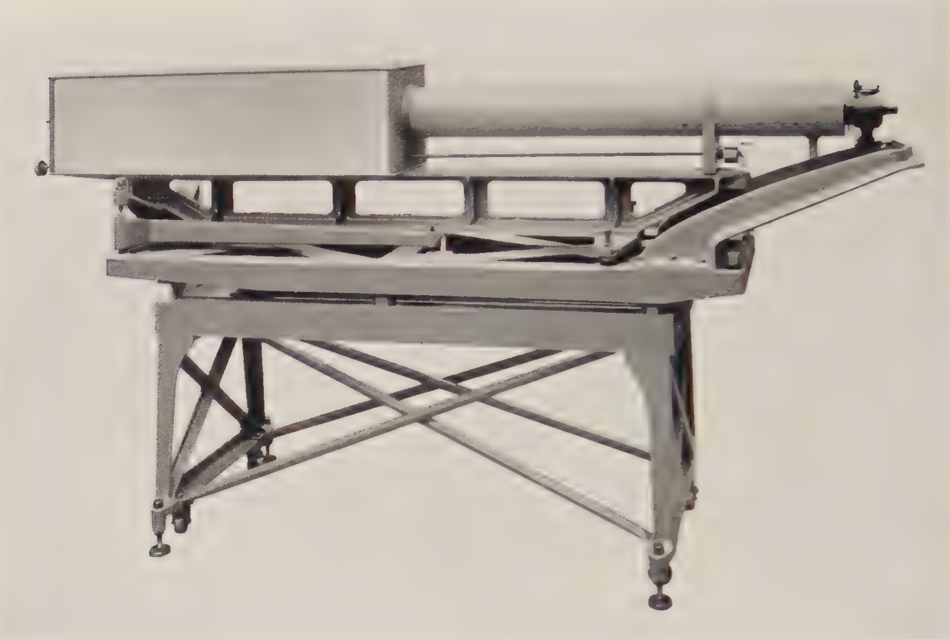

Figure *A*.

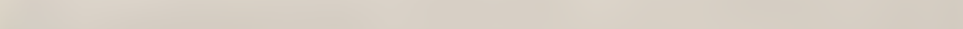

Figure *B*.

represents a wonderful technical achievement in lens-making. As homogeneous fluorspar of the required dimensions is not available, this component has to be constructed from a plate made up from four or five pieces, each of which has to be carefully cut to shape, and the pieces are cemented together to form a plate before being ground and polished to the lenticular form. The cement must be sufficiently rigid to prevent even a minute relative movement of the separate parts, while it must be sufficiently elastic to avoid accidental double refraction in the easily strained fluorspar. To yield critical definition the lens must be carefully centred, and adjustments for this purpose are included in the lens-holder.

The problem of mounting plates has been solved by using a substantial box of invar steel; they are held in position by four thin plates which touch them at the edges only, and a small amount of end play is allowed. The ends of the box are enlarged to carry two adjustable mirror strips, the purpose of which is explained in the next section. These ends also have four set-screws and, the framework being standardized, different echelons can be used in the same pressure chamber C. The pressure was originally measured by means of a tubular gauge containing a globule of mercury, but this tarnished in the course of time and tended to move jerkily when the pressure was uniformly varied. A pressure-gauge working on the principle of the aneroid barometer with a 360-degree scale corresponding to pressures of from 0 to 50 mm. has been substituted in the more recent instruments. An auxiliary pressure or vacuum chamber P, connected by means of a needle valve, allows the pressure in the echelon chamber to be varied without the whole having to be connected to a pumping system.

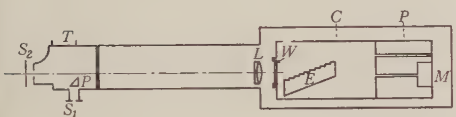

Figure 7.

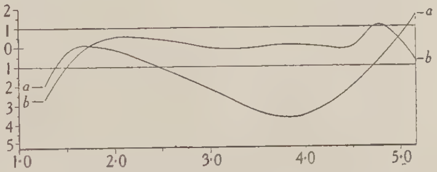

Figure 8. Residual chromatic aberration for $F = 1000$ mm. The ordinates give the variation of the focal length in mm. The abscissa is the corresponding wave number $\times 10^{-4}$.

In order that the full advantages of a reflection echelon may be obtained it should also be possible to use it so that its dispersion can both assist and oppose that of the auxiliary spectrograph. In the absence of another high-resolving-power instrument or another echelon of slightly different plate-thickness this forms the only available method of deciding whether a satellite lies on the long or short-wave-length side of a main line. For this reason the chamber unit is made in a cylindrical form and mounted on four rollers so that it can be rotated by means of an adjusting-rod, the pinion of which engages in a cogged wheel that forms part of the chamber. Friction brakes at the sides prevent the chamber from rotating too easily. The head carrying the slit and the reflecting-prism can be rotated so that the slit can be placed either above or below or at either side of the collimator tube, the exact positions being fixed by means of a spring plunger. This has proved a great convenience in practical work and it is unfortunate that it cannot be applied when a mirror mount is used.

If a relative movement of the two instruments can occur, owing to vibration, a corresponding loss in definition will be experienced. Any attempt to obtain the necessary rigidity by mounting the two instruments on piers on a common concrete foundation would limit their use according to the nature of the auxiliary apparatus needed in connexion with the light source, such as liquid-air or liquid-hydrogen cooling. If the base of the auxiliary mount pivots about a point immediately below S_2, figure 7, the mount can swing in any direction by a considerable amount without displacement of the fringes on the slit S_2. This method has been adopted. A rigid ball-and-socket bracket is clamped to the spectrograph, and one end of the base rests on this while the other two points of the support are slightly outside the centre of gravity of the whole system. The mounting carrying the collimator tube, lens, and echelon chamber consists of a rigid girder frame. This in turn rests on the top point of a vertical triangular casting, the base of which has two studs which fit into grooves in the base of the mount. The chamber end of the girder rests on two projections on a similar vertical casting to which a long horizontal arm is attached. The end of this arm is held to the base by a spring and screw. When the screw is turned, the girder carrying the optical parts moves horizontally either towards or away from the spectrograph. This adjustment, the design which is due to Mr Dowell, is necessary if a parallel beam is to fall on the echelon. The position of the lens L is then fixed by that of the first slit. The whole instrument can be moved by means of this adjustment so that the slit S_2 of the spectrograph is at the focal plane of the lens L. It has been found a great convenience to mount both the instruments on substantial trolleys with clamping bolts which are used when the instruments have been wheeled into position. It is experimentally found that long exposures of 10 or 15 hours for weak lines show nearly as good definition as the short exposures of 5 or 10 minutes required for the stronger lines in the mercury arc. The general appearance of the mounting can be seen from Figure *A* in the plate. Figure *B* shows a side view of the echelon and pressure chambers in position.

The question of the optimum value of the focal length of the lens is a matter of some difficulty. With the 170-cm. focal length the distance between the orders is about 1 mm. in the red, the separation becoming correspondingly smaller as the wave-length is decreased. It is highly desirable to use soft gradation plates, not only on account of their speed but also for the fact that faint satellites are more easily seen on them. Unfortunately such plates have large grain-size, and unless the separation of the orders is about 2 mm. this effect will influence the practical resolving-power obtained.

If only the more intense lines are to be examined, a lens with a 3-metre or even longer focal length might be employed. The intensity would be approximately four times weaker than with the 170-cm. objective, and many lines which can be photographed with the latter would become impossibly weak. The numerical aperture ratio of the E 1 spectrograph is in the neighbourhood of $f/30$, and is sufficiently low for most spectra when the high dispersion of the echelon is considered. It is suggested that when intense lines in the ultra-violet have to be examined with the maximum available resolving-power, a concave mounting on the lines described by Ritschl(13) can be employed. The astigmatism of the mirror becomes negligibly small for a large radius of curvature, since the necessary tilt is reduced.

The mounting described above appears to constitute the best compromise when light-intensity as well as accuracy has to be considered.

§ 8. WAVE-LENGTH DETERMINATIONS BY MEANS OF THE REFLECTION ECHELON

The general method has already been briefly described in a letter to *Nature*(15). Two platinized mirrors with adjustable surfaces are mounted, one on either side of the echelon stair. Their length is equal to the total width of the echelon steps and their breadth is about 5 mm. They are adjusted so that the pattern on the photographic plate, with a wide spectrograph slit and the echelon steps horizontal, is as shown diagrammatically in figure 9. A_1 and B_1 are respectively the $(m_1 - 1)$th and m_1th orders for a known wave-length λ_1. C_1 and D_1 are images of the primary slit S_1, figure 7, reflected by the above mentioned mirrors. Correspondingly A_2, B_2 are the orders for another known wave-length λ_2, while C_2, D_2 are the mirror images for this wave-length. Since the various mirror images correspond to identical points on the spectrograph slit S_2, the path difference of the light diffracted from the echelon to one of these points would be constant. The order of interference at C_1 would be $(m_1 + f_1)$, where f_1 is the fractional part represented by the ratio C_1B_1/B_1A_1. Similarly, that for λ_2 is given by the ratio C_2B_2/B_2A_2. If a line such as λ_3 is in single-order position the separation of the orders cannot be measured, but if the separation for another wave-length λ_1 is known it can be calculated since

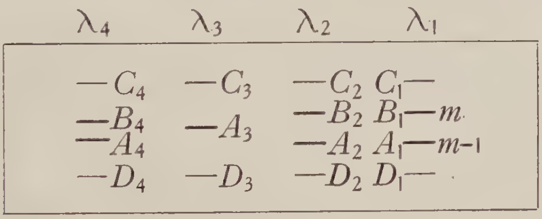

Figure 9.

f_1

$$\text{order separation for } \lambda_3 = A_1B_1\frac{\lambda_3}{\lambda_1} \times \frac{C_3D_3}{C_1D_1}$$

and the fractional part f_3 will be C_3A_3 divided by this quantity.

We then have $$(m_1 + f_1)\,\lambda_1 = (m_2 + f_2)\,\lambda_2 = (m_3 + f_3)\,\lambda_3,$$

and the accurate values of $f_1, f_2, f_3, \lambda_1, \lambda_2, \lambda_3$, and the approximate value of m being known, the exact value can be quickly found, and the product $(m_n + f_n)\lambda_n$ is determined to an accuracy depending on the accuracy with which the standard wave-length and its fractional part are known. If the wave-length of a fourth line λ_4 is sufficiently known, so that there is no ambiguity in the value of the integer m_4, a determination of f_4 will enable the wave-length to be obtained to a much higher degree of accuracy. Table 2 shows a typical measurement and calculation for a number of neon lines with a 25-plate echelon. These were made by Mr Rymer at this laboratory. He had had no previous experience in plate measurements.

The echelon chamber was evacuated, so that the vacuum wave-lengths of the neon lines had to be calculated from Meggers and Peters's formula. The vacuum wave-lengths of red cadmium has been taken as 6440·2513 Å., this being the provisional value recently determined by Sears and Barrell(16).

Table 2.

Wave-length in air	Wave-length in vacuo	Observed fractional part
6438 Cd	6440·2513	0·973
6717 Ne	6718·896	1·374
6598 Ne	6600·774	0·997
6532 Ne	6534·686	1·560
5975 Ne	5977·185	1·165

From the micrometer determination of the plate-thickness the order of interference for 6438 Å. is approximately 22630. Table 3 shows the various orders and fractional parts of the neon lines calculated for various orders of λ 6438.

Table 3.

Wave-length	6438	6717	6598	6532	5975
Observed	0·973	0·37$_4$*	0·99$_7$*	0·56$_0$*	0·16$_5$*
	22654·97	21715·43	22104·03	22327·58	24410·11
	22655·97	21716·38	22105·01	22328·56	24411·18
	·········	·········	·········	·········	·········
	22656·97	21717·34	22105·98	22329·55	24412·26

From the agreement of the calculated fractional parts of the middle row with the experimentally determined values given above it is evident that this is the correct order, so that the effective path difference for this instrument is 1·45910140 cm. This is very closely twice the thickness of the echelon plate.

Wave-length measurements with the reflection echelon in this manner appear to have very marked advantages over the corresponding determination with the Fabry-Perot interferometer. The primary advantage is that the resolving-power as measured by the sharpness of the fringes is independent of the wave-length, while the reflecting-power of the metallic films of the Fabry-Perot instrument decreases rapidly in the ultra-violet region. A practical advantage is that the determination of the fractional part can be made in much less time than the corresponding determination with the Fabry-Perot instrument; this is of importance when a large number of lines have to be measured. No account need be taken of the variation with wave-length of the phase-change at reflection, and the instrument remains in permanent adjustment. By making the determination in vacuo all the corrections for atmospheric temperature, pressure, humidity, and carbon-dioxide content of the air are eliminated, and the wave-numbers are obtained directly without recourse to any refractive-index tables.

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* The neon tube employed gave somewhat broad, unsymmetrical fringes due to an incomplete resolution of the isotope displacements. The measurements show that the inclusion of the third decimal for the fractional part is not justified in this instance and is only given as an indication.

This method enables the absolute wave-number of any satellite to be determined with somewhat better accuracy than the difference between its wave-number and that of its main line, since the latter may be somewhat broader while the reference points for the sharper satellite are the extremely sharp reference mirror images and the orders of the standard line.

A 40-plate echelon now under construction should be able to resolve two lines of equal intensity which have a separation of 0·018 cm^{-1} independently of their position in the spectrum. The wave-number difference corresponding to a whole order is 0·720 cm^{-1}, so that the wave-number of a sharp line should be obtained to within 0·001 or 0·002 cm^{-1}.

It is well known that owing to the character of the intensity-distribution curve in echelon spectra, the centres of gravity of the two orders of a somewhat broad line are spuriously displaced towards each other. Merton and Barratt⁽¹⁷⁾ have given a method depending on the measurement of the width of the line in single and double-order position whereby this displacement can be evaluated.

The following method will give this quantity more directly. If we write κ for the κ

$$\frac{\text{mirror image distance}}{\text{order distance} \times \text{wave-length}},$$

κ should be constant for all sharp lines on a given plate. This has been checked by measurements of sharp satellites from a water-cooled mercury lamp. A somewhat broad line of wave-length λ_b should have an order-separation $\kappa \times \lambda_b \times M_b$, where M_b M_b is the distance between its mirror images, which will be sharp. The actual separation between its orders will be less than this amount, and the difference between the two values will be twice the amount by which each line is apparently displaced. If this correction is made in the reading of the higher-order fringe, the fractional part obtained will be independent of the intensity-distribution of the echelon.

If a line is in single-order position no such displacement will occur, and in an intermediate position the two displacements should very nearly cancel. This can be seen by drawing an envelope curve $y = \sin \alpha/\alpha$ and setting up ordinates α at intervals π. Thus the provision of means for varying the pressure is almost superfluous. The echelon chamber can be kept permanently evacuated so that the constant κ remains unaltered.

The importance of the reflection echelon as an instrument both for standard wave-length measurements and for hyperfine-structure work lies in the fact that the resolving-power of a Fabry-Perot interferometer is in practice limited by reflecting power of the films. While in theory the power can be increased by increasing the separation of the plates, in practice this is often not practicable beyond a certain point owing to the consequent overlapping of components. There is little doubt that with the development of optical technique, reflection echelons of a hundred or more plates will become practical possibilities in the not too distant future. The patterns then observed will show the true intensity-distribution of the spectral line.

A Fabry-Perot etalon with fairly dense, freshly made silver films should give for red light approximately the same resolving-power as a 25-plate echelon having the same plate-thickness as the gap of the interferometer. This comparison has been made several times, and one is forced to the conclusion that the actual resolving-power of the echelon is approximately twice as great. The discrepancy could easily arise if the etalon films were not strictly plane and parallel, and therefore special care was taken to ensure that no detectable error should occur. The only explanation that can be tentatively offered is that the inevitable minute errors in the Fabry-Perot instrument are multiplied, so that the multiple reflections occurring after a total error of $\frac{1}{4}\lambda$ has been reached tend to reduce instead of increasing the sharpness of the fringes. The reflection echelon plates are individually made, so that the possibility of additive errors is eliminated.

§ 9. DETERMINATION OF THE LENGTH OF LENGTH STANDARDS BY MEANS OF THE REFLECTION ECHELON

Two methods have hitherto been available for the purpose of standardizing length units in terms of the wave-length of some standard line: (1) Michelson's method, which has recently been further developed by Kösters(8), who used a Twyman and Green interferometer; (2) Benoit's method, which has been improved upon by Sears and Barrell(16).

The source of error most difficult to eliminate is the spurious displacement of the fringes that would occur if the standard source were complex and unsymmetrical. The possibility of this error was realized by Michelson who showed that if a line was perfectly sharp on one side and slightly shaded on the other, the fringes in his interferometer would be displaced by as much as 0·12 of a fringe-width even when the path difference was so small that the clearness or visibility of the fringes was not very much impaired. This effect is clearly shown in figure 3 for the case of the green mercury radiation.

Because of multiple reflection, the second method gives a fringe which is a much truer picture of the intensity curve of the line itself than is that given by the first method, and the spurious displacement owing to any lack of symmetry should be much reduced. The comparative coincidence in the final results of the two methods on the metre determination would, apart from the possibility of other errors cancelling its effect, apparently indicate that the red cadmium source is symmetrical.

On the other hand Nagaoka(18), using a special field-free cadmium lamp, reports alternations in the visibility of the Fabry-Perot fringes as the separation of the plates is increased to 20 cm. Heydenburg(19), who has studied the Paschen-Back effect in hyperfine structure and the polarization of the resonance line, concludes that the separation of the odd isotope levels at 6^1P_1 is 0·0126 cm.$^{-1}$ with the stronger even isotopes in between, so that the complex line 6438 Å. should be symmetrical.

The error due to lack of symmetry will also be present in the method suggested below, but its magnitude can be shown to vary inversely as the effective number of reflections employed. A densely silvered Fabry-Perot interferometer should give an effective number of about 25, but since it has to be used in series with stepping-up etalons in the normal Benoit method such heavy silvering cannot be employed.

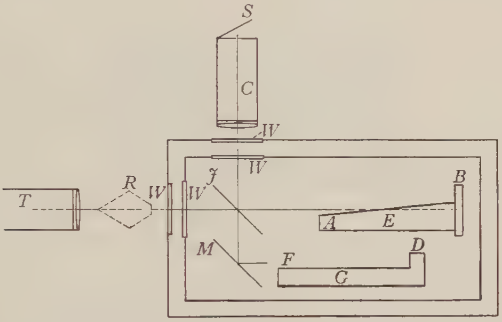

Figure 10.

Apart from any errors due to the multiplying or stepping-up process or errors in the determination of the phase-changes involved, there is the possibility of a change in the effective retardation of the primary etalon by the time it is being used in the stepping-up process. The principal advantage of Benoit's method, apart from the phase-displacement effect discussed above, lies in the very much reduced time taken to complete an observation, so that pressure and temperature conditions are fairly constant.

The method suggested below enables the determination of a length to be carried out in two operations which can be alternately checked in a few minutes. No corrections need be applied for phase-changes, and the limiting factor in accuracy seems to depend on the limits within which the temperature of the steel gauge can be determined. Finally, the adjustments required are so simple that the determination can be carried out in vacuo, and the complications of standardization in air can thus be avoided.

A silica reflection echelon of a total length equal to that of the distance to be standardized is made. If, for example, a metre gauge is desired, the plate-thickness may be 4·0000 cm. so that with a 25-plate instrument the distance between the ends will be a metre, within a few wave-lengths. The first plate is uncovered, and an extra large plane surface forms the last plate. This is now placed in one arm of a

Twyman and Green interferometer as shown in figure 10. For this purpose the cross slit S on the collimator C is closed, forming the required point source. The light transmitted by the half-silvered mirror J is reflected at M to the gauge G. When the surfaces of A and F, as well as B and D, are equidistant from J, the appearance of the field of view when the eye is placed at the focal plane of the objective of T will be as shown in figure 11 (A). If the distance between the planes A and B is exactly equal to that between the planes of F and D, the straight-line fringes of P and R, which come from B and D of figure 10, will be in alignment with the fringes in Q, which are due to F and A of figure 10. Without this equality there will be a displacement of the fringes which could be determined most conveniently by measuring the tilt of either of two equal parallel plates, one placed in each interferometer arm. If desired the echelon gap E, figure 11 (A), of the interferometer image can be eliminated by adding a suitable reversing prism R, figure 10.

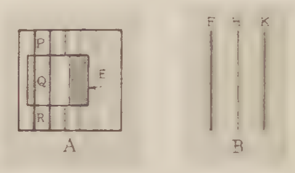

Figure 11.

The cross slit S is now tilted back and a micrometer eyepiece is added to the telescope T so that the appearance is as in figure 11 (B). H is the mirror image of the slit reflected from the open surfaces A or B. If the lines F and K are respectively the mth and $(m+1)$th orders for this wave-length, the exact order of interference at normal incidence is $(m - FH\ FK)$. The integer of m is determined as in the previous section, so that with a total of N plates $(m - f)\lambda = 2AB\ N$. Thus $(m - f)\lambda$ is the *mean* retardation due to a plate.

If the apparatus is enclosed in a double-walled box with suitable windows W the whole can be evacuated, and the tilting of the plate for equality of paths for E and G can be obtained by suitably geared remote control synchronous motors.

In this method one is not limited as to the choice of wave-lengths, since the resolving-power is not dependent on the reflecting-power. It might also be mentioned that the Twyman and Green or echelon fringes are entirely different from the fringes that have hitherto been used, and the echelon could be further used for the intercomparison of spectral lines throughout the spectrum to a degree of accuracy that has not yet been attained. The recent developments in the production of field-free sources seem to warrant the highest resolving-power.

§ 10. ACKNOWLEDGMENTS

I primarily wish to thank Prof. Takamine for his early encouragement and help, without which the first instrument could not have been constructed. I am indebted to Mr Rymer, B.A., of King's College for permission to include a set of his measure-

ments. Finally I wish to acknowledge the help of Mr Twyman, Mr Green and the technical staff of Messrs A. Hilger, Ltd., in connexion with the numerous difficulties that have had to be overcome.

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DISCUSSION

Mr C. V. Jackson. It is quite possible to use the reflection echelon for the determination of relative wave-lengths but it is not nearly so suitable for this purpose as the Fabry-Perot etalon, on account of the difficulty of getting sufficiently large dispersion. Indeed D. A. Jackson and myself considered the idea four years ago and came to the conclusion that the echelon could not be used for wave-length determinations of the highest accuracy for this reason.

When one is measuring the wave-lengths of sharp lines, it is easily possible to use an etalon 10 cm. thick, and if a projection lens of focal length 1 m. is used the dispersion is about 150 mm./Å.* at λ 5000. (The dispersion of course varies with the ring measured, but the above is about the dispersion one gets with the rings usually used for wave-length determinations.)

Now with a 25-plate echelon with plates $7\frac{1}{2}$ mm. thick and with a step of 1 mm., the dispersion in the same region is only about 5 mm./Å. with a 1·7-metre objective or 8 mm./Å. with a 3-metre objective.

Thus, to get the same dispersion as that obtained with a 10-cm. etalon and a 1-metre objective, it would be necessary to use a reflection echelon about 4 m. long, with an objective of focal length 3 m. It would not be feasible to use a greater focal length, as this would make the relative aperture too low.

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* This scale is necessary to obtain the highest accuracy attainable, since an error of 0·003 mm. (the minimum distance which can be reliably measured on a photographic plate) would give an error of 1 in 2×10^8 in the wave-length.

The determination of the phase correction of etalon plates presents no difficulty whatever. Indeed the data for its calculation are immediately at hand in the plates one takes, with different interferometer gaps, for measuring the wave-lengths.

It is quite true that the fringes obtained with an echelon have a smaller instrumental width than those obtained with an etalon, but this has no bearing on the accuracy obtainable in wave-length determinations, because one can always use an interference path sufficiently great to render the instrumental width small in comparison with the width of the line itself.

Although I have come to the conclusion, for the reasons given above, that the reflecting echelon is definitely unsuitable for the most accurate wave-length determinations, I do not wish to belittle an instrument for which I have a very great admiration. It is hardly overstating the case to say that as a high-resolving-power instrument for investigating hyperfine structure, it is far superior to any other instrument now known.

With regard to the silvering of interferometer plates, I think it necessary to point out that Dr Sears's etalon could not have been improved in any way. With the long interference path used by Dr Sears the reflecting-power of his silver was already high enough to give an instrumental width far narrower than the width of the Cd red line, or any other line known. The effect of heavier silvering would merely be to reduce the intensity of the rings. This is obviously disadvantageous.

Mr J. E. SEARS, Junr. The author's work on the reflection echelon has provided a powerful instrument of a new type which, I feel sure, will lead to interesting results.

It may be of interest to mention that we have only last week completed, at the National Physical Laboratory, a full redetermination of the length of the metre in terms of the cadmium red radiation, both in air free from CO_2 and in vacuo. The resulting wave-length in vacuo for λ_R is $0{\cdot}64402502\,\mu$., with a probable error so far as concerns the optical side of the measurements of less than 2 parts in 10^8. I mention this because the result is appreciably different from the figure quoted by Mr Williams, namely, $0{\cdot}64402513\,\mu$., taken from our preliminary measurements. The discrepancy is accounted for almost entirely by the fact that in the interval between the preliminary and final measurements the British national copy of the metre, No. 16, was re-verified at Sèvres, and its assigned value was reduced by 0·19 parts in 10^6.

The author mentions in his paper the possibility that the red line of cadmium has some fine structure, and I believe he has obtained definite evidence of this with the aid of his reflection echelon; I hope he will give us more information as to this as soon as he is in a position to do so. In the meantime, however, it may allay some doubts to record that in view of this possibility we carried out our work on the measurement of the metre starting with two independent basic etalons of length approximately $\frac{1}{12}$ m. and $\frac{1}{9}$ m. respectively, and found no measurable difference between the results.

The author mentions that owing to the necessity of using Brewster fringes for stepping up in the Benoit-Fabry-Perot procedure the plates of the primary etalon must be silvered fairly lightly, and he suggests that they will therefore not give as many as 25 reflections. Actually the plates used by us, which now appear somewhat tarnished after two months' continuous use, still give 18 visible reflections of a light-source such as the filament of an incandescent lamp. When new they probably gave between 20 and 25 reflections.

I am particularly interested in the author's suggestion that a reflection echelon should be used as a standard of length. It is a most attractive idea and has evident advantages, though it would be somewhat costly. From the metrological standpoint, however, there is one disadvantage that should be mentioned, namely that the gauge bar with a heel, with its two measuring surfaces facing the same way and not on the same axis, is obviously inconvenient. Some means might, however, be found to overcome this objection.

AUTHOR'S reply. In reply to Mr C. V. Jackson: I had understood that Mr D. A. Jackson had tried to use the reflection echelon as a means of measuring wave-lengths. The real reason of the apparent failure of the method is to be found not in any lack of dispersion but in the fact that with the Littrow type of mounting which he has adopted, the simple equation (2) does not apply and the more complicated equation (3) of § 7 has to be taken. This would necessitate measurements being made in two coordinates.

If spectral lines were simple and symmetrical a Michelson interferometer would suffice for the most accurate wave-length measurements, since the path-difference could be increased as required. Unfortunately very few if any such homogeneous lines exist, and it would only be in very rare instances that a Fabry-Perot etalon with a gap of 10 cm. could be used. This is shown from Mr C. V. Jackson's own work* on the wave-lengths of the Krypton lines which have been considered by various foreign metrologists to be extremely sharp. The largest gap he used was 3 cm., although a gap of 10 cm. was attainable with the instrument. In spite of its lower dispersion a 35-plate echelon of plate-thickness 7·3 mm. clearly shows fine structure in lines that the much-larger-gap etalon apparently fails to resolve. Surely if the dispersion suffices to show the components clearly separated it can be used with correspondingly increased accuracy to measure their individual wave-lengths. It is not the linear dispersion of an instrument that finally sets the limit to the accuracy of the wave-length measurement, but its resolving-power, provided this has not been attained at the expense of overlapping orders. All that is needed as regards dispersion is that it should be sufficient to eliminate any effects due to the grain of the plate. Many spectral lines (especially when emitted from a Paschen-Schuler hollow-cathode discharge) are so intense that a suitable achromatic lens could be placed in front of s_2, figure 7, to give a five-fold magnification of the pattern on the slit of the spectrograph. Although the dispersion would then be increased, the lines themselves would be correspondingly wider and the real gain in accuracy would be small.

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* *Proc. R. S.* **138**, 147 (1932).

With the etalon method one has to measure the diameters of comparatively large rings of diameter about 1·5 cm., and the slightest shrinkage of the photographic film will appreciably alter the apparent order at the centre. In the new method, the sharp reference line need only be a fraction of an order away from the interference fringe, so that the shrinkage effect would be negligible. The measurements carried out so far with reflection echelons of thickness 7 mm. indicate that if correspondingly sharp lines are obtained with a 25-plate instrument of plate-thickness 4 cm. (the construction of which is a practical possibility) an accuracy of 1 part in 10^8 would be attainable at a wave-length of 6000 Å.

I am much obliged to Mr Sears for his kind interest and am indebted to him for a wave-length standard with which to make intercomparisons. He mentions that his etalons gave about 25 multiple reflections. If we assume that the weakest image has an intensity 0·01 or 0·001 of the first, the approximate reflection coefficient of the films can be obtained, since $R^{25} = 0{\cdot}01$ or $0{\cdot}001$ as the case may be. The calculated values of the coefficients are 0·759 and 0·832 respectively. An inspection of Hansen's curve *A* given on p. 99 of my little booklet* shows that the corresponding effective numbers of reflections are approximately 11 and 16. That is to say, the etalon should be equivalent to an 11- or 16-plate reflection echelon of the same thickness. The real advantage of the reflection echelon is this: above a certain point high revolving-power with the Fabry-Perot interferometer can only be obtained by increasing the plate-separation, which increases the probability of overlapping of orders of satellites. In the reflection echelon high resolving-power is attained by simply adding extra plates.

* *Applications of Interferometry* (Methuen).

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OBSERVATIONS ON THE INTENSITY OF LOW-FREQUENCY SOUNDS CLOSE TO A METAL AIRSCREW

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ABSTRACT. A condenser transmitter and continuously recording cathode-ray tube have been used to obtain the wave-form of the sound from a metal airscrew operating at zero rate of advance. Fourier analysis has been applied to records taken at various distances up to 300 ft. from the airscrew and the rate of decay of intensity has been determined for the first three harmonics. At distances greater than 100 ft. the inverse-square law of distance has been found to hold, but at nearer points the rate of decay is not constant and varies in a complicated manner. The average rate of decay is less than that given by the inverse-square law, a result not in concordance with recent determinations by Obata, Yosida and Morita. An explanation of this difference is given.

1. INTRODUCTION

An interesting property of the sound generated by a complex source, such as a revolving airscrew, is the rate of decay of intensity of the low-frequency components at distances which are not great compared with the dimensions of the source or with the various wave-lengths. If the sound from the airscrew can be isolated from the sound emanating from the power plant, a convenient method of obtaining this information is to apply Fourier analysis to wave-form records of pressure-amplitude. The results so derived may be of value for the purpose of testing proposed theories concerning the airscrew as a source of sound. The wave-forms themselves are of considerable interest, inasmuch as they provide material for comparing the physical magnitudes of the high-pitched sounds, which are so noticeable in the neighbourhood of an airscrew, with those of the low-pitched sounds which, on occasion, are felt rather than heard. Furthermore, Eisner, Rehm and Schuchmann(1) have pointed out that the search-tone methods of frequency-analysis sometimes employed in the analysis of aeroplane noise give inaccurate results for components having frequencies of less than 100 ~, and that more detailed knowledge of the intensity of these components is required, especially close to the aeroplane. It is hoped that the results given in this paper may assist in the collection of this information so far as the airscrew sound is concerned.

2. EQUIPMENT AND SCOPE OF EXPERIMENT

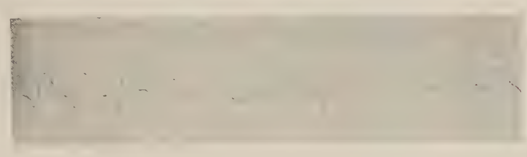
The airscrew used was of the Leitner-Watts type with hollow steel blades; it had a diameter of 11·5 ft. and a pitch setting of − 5 degrees against the centre scale. The power plant was a Napier-Lion engine installed in a tethered wingless fuselage (D.H. 9A type), which was adjusted in height so that the plane of rotation of the airscrew was vertical, the tips of the blades clearing the ground by 2 ft. As in previous experiments conducted by the author(2), this equipment was set up remote from buildings, in order to avoid errors due to the reflection of sound therefrom. The sound from the engine exhaust was effectively silenced by attaching to the cylinder stub pipes lagged manifolds terminating in a silencing pit.

The sound from the airscrew was received by a calibrated condenser transmitter (G.E.C. pattern) placed with the diaphragm almost flush with the ground. In this way, the interference effect of the waves reflected once from the ground between the airscrew and the transmitter was minimized. The sound-currents were amplified once and passed over a low impedance line to the laboratory, where, after further controlled amplification, they actuated a Von Ardenne cathode-ray-tube, the wave-form being recorded on a strip of sensitized paper moving at uniform speed.

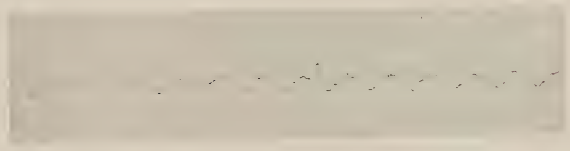
Observations were made in the plane of rotation at the following horizontal distances (in feet) from the airscrew boss: 300, 100, 50, 40, 30, 20 and 10. The plane of rotation was chosen because earlier experiments had shown the sound-output to be particularly steady in this direction. To record the wave-form, the engine was first allowed to attain a speed of 1700 ± 10 r.p.m. A calibrated step-by-step amplification-control was then set so that the maximum excursion of the cathode-ray spot was such that the photographic record would have an amplitude adequate for purposes of analysis and inspection. The duration of each recording period was a few seconds.

3. RESULTS

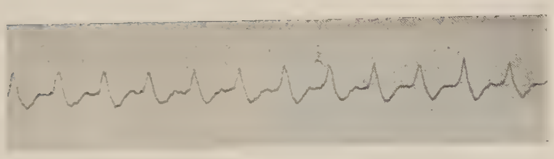
Figure 1 comprises reproductions of typical wave-form records. By inspection, successive waves on any given record are sufficiently alike for Fourier analysis to be applied with accuracy in the computation of the lower-order harmonics. This repetition of wave form has an important bearing on the effectiveness of the exhaust-silencing. The fundamental frequency in the sound from the airscrew is clearly 37·3 ~ for an engine speed of 1700 r.p.m. The Lion engine has a reduction gear of 1 to 0·659, so that the fundamental frequency of the exhaust sound is 56·7 ~. With few exceptions the frequencies comprising the two trains of harmonics are, in effect, incommensurable, and hence repetition of wave-form could not occur if the exhaust sound were present in any appreciable quantity. The period of repetition of the waves is the period of the fundamental blade-frequency. The method of silencing can therefore be regarded as adequate. Inspection also shows the maximum amplitude of any wave-trace to be very steady, and not only was this steadiness evident throughout the length of each record, but the amplitude of the cathode-ray spot-displacement was remarkably constant over long periods when the engine had attained constant speed. This may be characteristic of metal airscrews, for both two-bladed and four-bladed wooden airscrews, when driven by the same engine under similar weather conditions, produced amplitudes and wave-forms of very appreciably less constant nature.



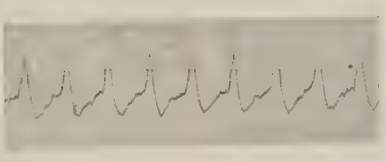
At 300 ft.



At 50·5 ft.



At 21·4 ft.



At 12·6 ft.

Figure 1. Specimen oscillograms of airscrew sound.

Figure 2 incorporates the results of Fourier analysis after correction for the overall frequency characteristic of the recording apparatus. This characteristic was sensibly flat from 55 ~ upwards, the presence of two line transformers accounting for some attenuation at lower frequencies. For the purpose of analysis, the ordinates of six successive waves were measured on each record. These were averaged to give a mean wave-form which was then analysed. In this way, small variations in wave-form were smoothed and the effect of any residual frequencies, not exact multiples of the fundamental blade frequency, was minimized by interference. The two sets of results given in the figure were obtained on different days and correspond to engine-speeds of 1700 r.p.m. and 1750 r.p.m. The values of the pressure-amplitude also are tabulated.

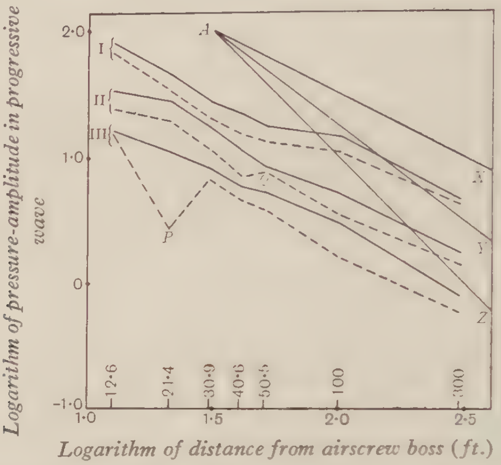


Figure 2. Experimental relationship between pressure-amplitude and distance from the centre of the airscrew. Laws of intensity: *AX* inverse-square, *AY* inverse-cube, *AZ* inverse-fourth-power. Harmonics: I first, II second, III third. Engine-speeds: 1750 r.p.m. ——, 1700 r.p.m. - - - - - .

Table 1.

Engine-speed (r.p.m.)	Frequency	Pressure-amplitude (dyne/cm.^{2}) in progressive wave at the following distances from the boss (ft.)						
		300	100	50·5 (50)	40·6 (40)	30·9 (30)	21·4 (20)	12·6 (10)
1700	37·3 74·6 111·9	4·3 1·4 0·6	11·2 3·5 1·6	13·3 7·7 3·9	15·4 7·1 4·7	20·6 11·6 6·8	34·6 19·7 2·8	68·4 24·8 15·8
1750	38·8 77·6 116·4	4·7 1·8 0·8	14·9 5·3 3·1	17·6 8·6 5·3	22·4 11·7 6·0	27·8 18·0 8·3	46·0 28·2 11·1	81·6 34·4 16·0

The figures in brackets are horizontal distances from the airscrew boss.

The two sets of results are in agreement so far as the general form of the unsmoothed curves is concerned. No significance is attached to the departure of the two plotted points *P* and *Q* from the general trend of results, such occasional discrepancies being inseparable from experiments carried out in the open air. At distances greater than 100 ft. the intensity falls off according to the inverse-square law of distance, as might be expected, since distances of this order are large compared with the dimensions of the source and with the wave-length. The greater rate of decay indicated for the third harmonic (at 1750 r.p.m.) is probably erroneous and due to a slight over-estimation of the pressure-amplitude at 100 ft., the logarithmic scale causing a relatively large change in the position of the plotted point. At nearer distances are found various rates of decay for which no reason can be advanced at present. Thus, the fundamental component (first harmonic) has an abnormally slow rate of decay between 50 ft. and 100 ft., but at lesser distances both the inverse-square and inverse-cube laws of distance are followed. It should be noted that the abscissae in figure 2 are the distances of the microphone from the airscrew boss. Had the horizontal distances from the boss been plotted, the inverse-square law alone would have applied approximately between 10 ft. and 50 ft., as may be seen by reference to the pressure-amplitude values given in the table. According to both sets of results the second harmonic follows the inverse-cube law between 21·4 ft. and about 40 ft., but very close to the airscrew it decays at a lesser rate than the inverse-square. Further, a comparison of the results obtained at 300 ft. with those obtained at the nearest microphone-position show that the average rate of decay of intensity is less than that corresponding to the inverse-square law of distance, whether the distance is measured from the airscrew boss or from a point on the ground vertically beneath the boss.

These observations are apparently at variance with those recently put forward by Obata, Yosida and Morita(3). Experiments in which an audiometer was used showed that the loudness of the sound from an airscrew varies inversely as the distance, within the limits of experimental error, and it was stated that this corresponds to a rate of decay of intensity much in excess of the inverse-square law of distance. In an obvious notation, this result may be put in the form

$$db = -m\,(d_2 - d_1)$$

db, m, d_1, d_2

where m is in decibels per unit distance.

Assuming that the loudness of a complex sound may be converted into terms of physical intensity by the expression known to hold for pure tones, then if $I \propto d^{-n}$ it follows that

I, n

$$n = \frac{m}{10}\,\frac{d_2 - d_1}{\log d_2 - \log d_1}.$$

For distances between 30 and 100 metres $m = 0{\cdot}224$, giving $n = 3$, in correspondence with the inverse-cube law. Apart however from the above assumption, which is open to serious dispute, it must be remembered that a doubling and a halving of the physical intensity both produce a change in loudness of only $3db$ and that errors of this order are common in audiometer determinations, especially those made in the open. It does not seem possible, therefore, that adequate deductions concerning the rate of decay of intensity can be made from observations of a physiological character.

The specimen records shown in figure 1 indicate that no very marked change in wave-form occurs as the airscrew is approached. At first sight the wave-form at 300 ft. appears different from the others, but actually it is merely a reversed and inverted version, due to an inadvertent crossing of the leads between the line transformers. The wave-form at 300 ft. is smooth, but superimposed wavelets of comparatively high frequency become increasingly noticeable as the distance is diminished. Even at the nearest point to the airscrew, the average pressure-amplitude of these disturbances is only about 4 per cent of the maximum, an interesting fact in view of the impression made on the ear at such a close range.

4. ACKNOWLEDGMENTS

This experiment, which was carried out in connexion with the work of the Aircraft Noise Sub-Committe of the Aeronautical Research Committee, was initiated by Dr W. S. Tucker, D.Sc., O.B.E. The airscrew equipment was supplied for this and similar experiments by the Director of Scientific Research, Air Ministry.

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(1) Eisner, F., Rehm, H. and Schuchmann, H. *Elektrische Nachrichten-Technik*, **9**, 323–33 (1932).

(2) Kemp, C. F. B. *Proc. Phys. Soc.* **44**, 151–65 (1932).

(3) Obata, J., Yosida, Y. and Morita, S. *Aero. Research Inst., Tokyo Imperial University, Report No.* 79 (1932).

DISCUSSION

Mr J. Guild. In order to appreciate the physical significance of the author's results it would be necessary to have information which, while it may be quite well known to aeronautical experts, is not given in the paper. One point of importance is: how is the sound produced? Are we here concerned with a pure siren action due to the alternate arrest and release of portions of the air flowing through the plane of the screw (that is, can we regard the screw itself as a perfectly rigid body with no movement other than that of rotation?), or does the sound arise mainly or in any appreciable proportion from pulsations set up in the blades, which then act as vibrating bars? If the latter cause is responsible, what are the modes of vibration? Without an answer to these questions we cannot get at the phase and intensity relations which determine the form of the wave-front in the immediate neighbourhood of the source.

It is also necessary that we should know the distribution of air-velocity along the path followed by the waves; but no mention of air-velocities, which must be considerable in the vicinity of an aeroplane propeller, occurs in the paper.

The inclusion in technical papers of the information necessary to enable the results to be related to the physical conditions of the problem, though it would not add to the technical value of the results, would greatly increase their scientific value as additions to the general body of knowledge.

Dr A. B. WOOD enquired whether the author found any interference between sounds reaching the detector directly and by reflection from the ground. In connexion with § 1 he pointed out that the intensity of a sound depends on pitch as well as amplitude.

AUTHOR'S reply. In reply to Mr Guild: (1) The low-frequency sound here considered arises from the periodic impact of the airscrew blades upon the air in the path of rotation*. (2) Although elastic vibrations of the blades are present†, they constitute only a minor source of sound. (3) Variations in air-velocity may be disregarded, since the measurements were made in the plane of rotation, outside the slipstream (see § 2). (4) Existing knowledge of the sound generated by airscrews was discussed in a previous paper by the author‡ and a list of references was then given.

In reply to Dr A. B. Wood: (1) As was stated in § 2, the diaphragm of the microphone was almost flush with the ground. The recorded wave-form was therefore that of the directly received sound alone. (2) The intensity is given by $I = p^2/r$ where p is the r.m.s. value of the pressure-variation and r the radiation resistance of the medium. The quantity measured in the experiments was p, hence pitch does not enter into the calculations.

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* E. J. Lynam and H. A. Webb, *Aero. Research Committee R. and M., No.* 624 (1919); M. D. Hart, *Aero. Research Committee R. and M., No.* 1310 (1930).

† A. Fage, *Proc. R. S.* A., **107**, 456–68 (1925).

‡ C. F. B. Kemp, *Proc. Phys. Soc.* **44**, 151–65 (1932).

535·337 : 546·295

PRESSURE EFFECTS IN THE SPECTRA Xe I AND Xe II

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Communicated by Prof. A. Fowler, F.R.S., May 30, 1933.
Read July 7, 1933.

ABSTRACT. The first and second spectra of xenon have been examined for pressure effects. At high pressure, lines of the Xe I spectrum show broadenings which correspond qualitatively to their respective Stark effects. This points to the existence of strong interionic electric fields within the discharge. Many pressure displacements in the Xe II spectrum are also found, and it is suggested that these too have their origin in interionic fields.

§ 1. INTRODUCTION

Numerous investigations of the broadening and displacement of spectral lines when excited under conditions of high current-densities and high vapour-pressures have been made. In the main these investigations have supported the view that such broadenings and displacements are, to some extent at least, a result of interionic electric fields. In arc spectra it has often been possible to correlate qualitatively the broadening and displacement with the known Stark effects. Thus, for example, Merton and Nicholson* have shown that the symmetrical broadening of the Balmer lines in a condensed discharge is that to be expected as a result of inhomogeneous electric fields; and Merton† and Takamine‡ have demonstrated a corresponding effect for helium lines excited by a condensed discharge or in gas at high pressure. So-called pressure shifts are also known in the second spectra of many elements. Pretty§ has made systematic investigations of these shifts in the spectra of ionized nitrogen, oxygen, neon and argon, and found reasons to believe that they arose from electric fields, although actual Stark effect data were not available for comparison.

In the xenon arc spectrum Humphreys‖ reported that he was unable to discover any shift of the lines with varying pressure of the gas, but that under condensed discharge conditions he observed some asymmetric broadenings toward the red. These he attributed to the effect of electric fields. Because Stark displacements are now known¶ to be unusually large and abundant in the xenon arc spectrum, it was of interest to the writer to investigate the effects of pressures higher than those used by Humphreys. The results show that broadened lines are emitted by an uncondensed discharge in the gas at pressures of several centimetres of mercury and that these broadenings are to be attributed to the effect of interionic electric fields. The pressure effects in the first spark spectrum were also investigated and numerous shifts are reported and discussed.

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* *Phil. Trans.* **216** A, 459 (1917).

† *Proc. R.S.* A. **95**, 30 (1918).

‡ *Sci. Papers of Inst. of Phys. and Chem. Research* (Tokio), No. 69, 55 (1926).

§ *Proc. Phys. Soc.* **41**, 442 (1929); **43**, 279 (1931).

‖ *Bureau of Standards J. of Research*, **5**, 1041 (1931).

¶ H. W. Harkness and J. F. Heard, *Proc. R. S.* A, **139**, 416 (1933).

§ 2. EXPERIMENTAL

The sources of light were pyrex discharge tubes of the Geissler type which could be viewed end on. They were fitted with aluminium and magnesium electrodes, the magnesium serving to clean up impurities. Into one such discharge tube xenon was admitted to a pressure of about 4 mm. and into another to a pressure of about 4 cm., the latter pressure having been found to be the most suitable for a high-pressure source.

A 12-in. induction coil and a 40-kV. transformer were used as means of excitation. When a condensed discharge was required a 0·07-μF. condenser and a 2 mm. spark gap were included in the circuit.

The light was examined by means of a Hilger E1 quartz spectrograph and a 10-ft. concave diffraction grating in an Eagle mounting.

§ 3. BROADENED LINES IN THE Xe I SPECTRUM

The spectrum of the condensed discharge in the low-pressure source showed the asymmetric broadenings mentioned by Humphreys*. It was discovered, moreover, that similar broadened lines could be produced by an uncondensed discharge in the 4-cm. source. In this case the transformer was used. The potential drop across the tube was about 15,000 V. and the current was about 25 mA. Under such conditions the discharge was disruptive and, in addition to the Xe I spectrum, the Xe II spectrum was excited though not strongly.

To say that the lines were broadened is, perhaps, less accurate than to say that certain lines were accompanied by wings on one side or both sides. A comparison of these with the known Stark displacements of the lines has shown conclusively that the wings arise from heterogeneous fields within the discharge. Thus, lines such as λ 6498·71, λ 5894·99, λ 5875·02, which have Stark displacements toward the blue only, have wings on the blue side but are perfectly sharp on the red side; lines such as λ 6198·25 and λ 6179·66, which have Stark displacements toward the red only, are diffuse on the red and sharp on the blue side; while lines which have Stark displacements on both sides show a distribution of intensity corresponding to the displacements and intensities of the Stark components. λ 6318·08 ($2p_8$–$6d'_4$) is an example of the latter type. The correspondence between its pressure broadening and its Stark effect is demonstrated in (*a*), (*b*) and (*c*) of the accompanying plate. A calculation indicates that some of the transitions responsible for the wings of this line must have taken place in the presence of electric fields certainly greater than 100,000 V./cm. Since the terminal potential of the tube was only 15,000 V., and since none of the observed light came from the neighbourhood of the electrodes, it must be assumed that the electric fields were of an interionic nature.

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* *Loc. cit.*

Note. The following observation, although divorced from the present subject of discussion, is probably worth reporting here. When the high-pressure xenon discharge tube was excited by a weak discharge the greater part of the visible light consisted of the Swan band system of C_2. The appearance of these bands was very similar to the appearance of the Swan bands excited by Johnson*, using high-pressure argon. The mechanism is, no doubt, of a similar nature, for in the present case the discharge tubes contained traces of carbon and hydrogen as in Johnson's experiments.

§ 4. BROADENED AND DISPLACED LINES IN THE Xe II SPECTRUM

The first spark spectrum of xenon as excited by a condensed discharge in the tube at 4 cm. has been compared with that excited in a similar way in the low-pressure tube. A 0·07-μF. condenser was shunted across an induction coil and a 2-mm. spark gap was placed in series with the tube. The spectral region investigated was that between λ 3500 and λ 7000.

All the Xe II lines excited in the high-pressure source are quite diffuse, and associated with the spectrum is a continuous background which makes it impossible to observe lines having less than a certain intensity. Of the lines observed many show displacements of their centres of gravity. Most of the displacements, including all the larger ones, are toward the red. The breadth of a line increases with the magnitude of its displacement, and consequently it is difficult to measure the displacements with precision; but an attempt has been made to estimate as well as possible the displacements of the centres of gravity. In the plate there is reproduced a portion of the spectrum showing some typical displacements of Xe II lines.

A partial analysis of the Xe II spectrum has been published by Humphreys, de Bruin and Meggers†. Many of their identified lines have been observed in the high-pressure spectrum. Table 1 lists these with the classification of the above authors and with the shifts observed in the present investigation.

From the observations in table 1 it has been possible to assign displacements to the energy levels involved in the various transitions. In general, each level is involved in several of the observed lines. The agreement is, naturally, not perfect, but it is within about 0·2 cm^{-1}. For this reason the small displacements have been recorded only as multiples of 0·3 cm^{-1}.

The displacements of the lines reveal not the absolute displacements of the levels involved but merely their relative displacements. In recording the displacements in table 2, the writer has chosen that arrangement which assigns zero displacement to the greatest number of levels.

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* *Phil. Trans.* **226**, 157 (1927).

† *Bureau of Standards J. of Research*, **6**, 287 (1931).

Table 1. Identified lines in the Xe II spectrum with their pressure displacements

Wave-length*	Classification	Dis-place-ment (cm.$^{-1}$)	Wave-length*	Classification	Dis-place-ment (cm.$^{-1}$)
6790·34	$(6s)\ {}^2D_{1\frac{1}{2}}-(6p)\ {}^2D^0_{1\frac{1}{2}}$	– 0·5	5368·07	$(5d)\ X_{1\frac{1}{2}}-(6p)\ {}^2P^0_{\frac{1}{2}}$	0
6788·60	$(6s)\ {}^2D_{2\frac{1}{2}}-(6p)\ {}^2D^0_{1\frac{1}{2}}$	– 0·6	5313·89	$(6p)\ {}^4D^0_{3\frac{1}{2}}-(7s)\ {}^4P_{2\frac{1}{2}}$	– 5·2
6694·35	$(6s)\ {}^4P_{\frac{1}{2}}\ -(6p)\ {}^4S^0_{1\frac{1}{2}}$	– 0·4	5309·28	$(6s)\ {}^2P_{1\frac{1}{2}}-(6p)\ {}^2D^0_{1\frac{1}{2}}$	0
6598·80	$(6s)\ {}^2D_{1\frac{1}{2}}-(6p)\ {}^2D^0_{2\frac{1}{2}}$	– 0·6	5292·22	$(6s)\ {}^4P_{2\frac{1}{2}}-(6p)\ {}^4P_{2\frac{1}{2}}$	+ 0·4
6597·24	$(6s)\ {}^2D_{2\frac{1}{2}}-(6p)\ {}^2D^0_{2\frac{1}{2}}$	– 0·6	5260·44	$(5d)\ X_{1\frac{1}{2}}-(6p)\ {}^2P^0_{1\frac{1}{2}}$	0
6343·97	$(5d)\ {}^4D_{1\frac{1}{2}}-(6p)\ {}^4S^0_{1\frac{1}{2}}$	0	5191·38	$(5d)\ X_{1\frac{1}{2}}-(6p)\ {}^2S^0_{\frac{1}{2}}$	0
6277·54	$(5d)\ {}^4D_{1\frac{1}{2}}-(6p)\ {}^4P^0_{2\frac{1}{2}}$	0	5122·44	$(6p)\ {}^4P^0_{\frac{1}{2}}\ -(7s)\ {}^4P_{1\frac{1}{2}}$	– 6·2
6097·60	$(5d)\ {}^4D_{2\frac{1}{2}}-(6p)\ {}^4S^0_{1\frac{1}{2}}$	– 0·6	5080·66	$(6p)\ {}^4P^0_{1\frac{1}{2}}-(7s)\ {}^4P_{1\frac{1}{2}}$	– 5·1
6093·53	$(6p)\ {}^4D^0_{1\frac{1}{2}}-(7s)\ {}^4P_{1\frac{1}{2}}$	large (–)	4988·78	$(6s)\ {}^2P_{\frac{1}{2}}\ -(6p)\ {}^2P^0_{1\frac{1}{2}}$	– 1·2
6051·18	$(5d)\ {}^4D_{3\frac{1}{2}}-(6p)\ {}^4P^0_{2\frac{1}{2}}$	– 0·9	4919·68	$(5d)\ X_{1\frac{1}{2}}-(6p)\ {}^2P^0_{1\frac{1}{2}}$	0
6036·20	$(5d)\ {}^4D_{2\frac{1}{2}}-(6p)\ {}^4P^0_{2\frac{1}{2}}$	– 0·5	4890·10	$(6s)\ {}^4P_{2\frac{1}{2}}-(6p)\ {}^4P^0_{1\frac{1}{2}}$	0
5976·47	$(6s)\ {}^4P_{1\frac{1}{2}}-(6p)\ {}^4S^0_{1\frac{1}{2}}$	+ 0·5	4862·50	$(6p)\ {}^4P^0_{2\frac{1}{2}}-(7s)\ {}^4P_{2\frac{1}{2}}$	– 5·8
5945·54	$(6s)\ {}^4P_{\frac{1}{2}}\ -(6p)\ {}^4P^0_{\frac{1}{2}}$	– 0·3	4823·38	$(6p)\ {}^4S^0_{1\frac{1}{2}}-(7s)\ {}^4P_{2\frac{1}{2}}$	– 5·7
5917·45	$(6s)\ {}^4P_{1\frac{1}{2}}-(6p)\ {}^4P^0_{2\frac{1}{2}}$	0	4818·04	$(5d)\ {}^4D_{1\frac{1}{2}}-(6p)\ {}^4D^0_{1\frac{1}{2}}$	– 0·7
5776·40	$(5d)\ X_{\frac{1}{2}}\ -(6p)\ {}^2D^0_{1\frac{1}{2}}$	0	4779·18	$(sp^6)\ {}^2S_{\frac{1}{2}}\ -(6p)\ {}^4S^0_{1\frac{1}{2}}$	– 0·7
5751·05	$(6s)\ {}^2P_{\frac{1}{2}}\ -(6p)\ {}^2D^0_{1\frac{1}{2}}$	– 1·3	4674·55	$(5d)\ {}^4D_{2\frac{1}{2}}-(6p)\ {}^4D^0_{1\frac{1}{2}}$	– 0·5
5719·62	$(5d)\ {}^4D_{1\frac{1}{2}}-(6p)\ {}^4P^0_{1\frac{1}{2}}$	– 0·3	4653·00	$(6s)\ {}^2P_{1\frac{1}{2}}-(6p)\ {}^2P^0_{1\frac{1}{2}}$	0
5667·58	$(5d)\ {}^4D_{1\frac{1}{2}}-(6p)\ {}^4P^0_{\frac{1}{2}}$	– 0·5	4603·05	$(6s)\ {}^4P_{1\frac{1}{2}}-(6p)\ {}^4D^0_{1\frac{1}{2}}$	0
5659·40	$(5d)\ X_{1\frac{1}{2}}-(6p)\ {}^2D^0_{1\frac{1}{2}}$	0	4384·92	$(sp^6)\ {}^2S_{\frac{1}{2}}\ -(6p)\ {}^4P^0_{\frac{1}{2}}$	– 0·9
5572·21	$(5d)\ X_{\frac{1}{2}}\ -(6p)\ {}^2S^0_{\frac{1}{2}}$	0	4215·63	$(6s)\ {}^4P_{2\frac{1}{2}}-(6p)\ {}^4D^0_{1\frac{1}{2}}$	0
5472·62	$(5d)\ {}^4D_{3\frac{1}{2}}-(6p)\ {}^4D^0_{3\frac{1}{2}}$	– 0·9	4100·34	$(5d)\ {}^4D_{1\frac{1}{2}}-(6p)\ {}^4D^0_{\frac{1}{2}}$	0
5460·40	$(5d)\ {}^4D_{2\frac{1}{2}}-(6p)\ {}^4D^0_{3\frac{1}{2}}$	– 0·9	3905·89	$(5d)\ {}^4D_{1\frac{1}{2}}-(6p)\ {}^2D^0_{1\frac{1}{2}}$	0
5450·49	$(6s)\ {}^2P_{\frac{1}{2}}\ -(6p)\ {}^2P^0_{\frac{1}{2}}$	– 1·3	3858·53	$(sp^6)\ {}^2S_{\frac{1}{2}}\ -(6p)\ {}^4D^0_{1\frac{1}{2}}$	– 1·0
5438·98	$(6p)\ {}^4D^0_{1\frac{1}{2}}-(7s)\ {}^4P_{\frac{1}{2}}$	0	3848·60	$(5d)\ {}^4D_{\frac{1}{2}}\ -(6p)\ {}^2D^0_{1\frac{1}{2}}$	0
5419·17	$(6s)\ {}^4P_{1\frac{1}{2}}-(6p)\ {}^4P^0_{1\frac{1}{2}}$	0	3811·06	$(5d)\ {}^4D_{2\frac{1}{2}}-(6p)\ {}^2D^0_{1\frac{1}{2}}$	– 0·7
5372·41	$(6s)\ {}^4P_{1\frac{1}{2}}-(6p)\ {}^4P^0_{\frac{1}{2}}$	0	3500·37	$(6s)\ {}^4P_{2\frac{1}{2}}-(6p)\ {}^2D^0_{1\frac{1}{2}}$	0

* The wave-lengths are those given by Humphreys, de Bruin and Meggers and are the wave-lengths in vacuo.

It will be observed that the only large displacements in table 2 are associated with two of the 7*s* levels. This fact is in agreement with the results of Pretty* on the corresponding A II and Ne II spectra, where the larger displacements are associated with terms of higher principal quantum numbers. In Pretty's results, however, members of a multiplet always give rise to displacements which are equal within the possible errors of measurement, whereas in the Xe II spectrum this is not the case. The most notable exception to such a rule lies in the smallness of the displacement of the term $(s^2p^4.7s)\,^4P_{\frac{1}{2}}$, as compared with the displacements of the other two members of the multiplet.

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* *Loc. cit.*

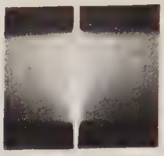
Table 2. Pressure displacements of the known energy levels of Xe II

Notation	Term value	Displacement (cm.$^{-1}$)	Notation	Term value	Displacement (cm.$^{-1}$)
(sp^6) $^2S_{\frac{1}{2}}$	80,195	− 0·6	$^4P^0_{2\frac{1}{2}}$	59,110	0
$(s^2p^4.6s)$ $^4P_{2\frac{1}{2}}$	78,000	+ 0·3	$^4P^0_{1\frac{1}{2}}$	57,556	+ 0·3
$^4P_{1\frac{1}{2}}$	76,004	+ 0·3	$^4P^0_{\frac{1}{2}}$	57,396	+ 0·3
$^4P_{\frac{1}{2}}$	74,210	0	$^4D^0_{3\frac{1}{2}}$	57,363	+ 0·3
$(s^2p^4.5d)$ $^4D_{3\frac{1}{2}}$	75,631	− 0·6	$^4D^0_{1\frac{1}{2}}$	54,286	+ 0·3
$^4D_{2\frac{1}{2}}$	75,672	− 0·6	$^4D^0_{\frac{1}{2}}$	50,654	0
$^4D_{1\frac{1}{2}}$	75,035	0	$^2D^0_{2\frac{1}{2}}$	49,012	0
$^4D_{\frac{1}{2}}$	75,416	0	$^2D^0_{1\frac{1}{2}}$	49,440	0
$(s^2p^4.6s)$ $^2P_{1\frac{1}{2}}$	68,270	0	$^2P^0_{1\frac{1}{2}}$	46,784	0
$^2P_{\frac{1}{2}}$	66,838	− 1·2	$^2P^0_{\frac{1}{2}}$	48,481	0
$(s^2p^4.5d)$ $X_{1\frac{1}{2}}$	67,105	0	$^2S^0_{\frac{1}{2}}$	47,847	0
$X_{\frac{1}{2}}$	65,788	0	$(s^2p^4.7s)$ $^4P_{2\frac{1}{2}}$	38,550	+ 5·7
$(s^2p^4.6s)$ $^2D_{2\frac{1}{2}}$	64,166	− 0·6	$^4P_{1\frac{1}{2}}$	37,879	+ 5·4
$^2D_{1\frac{1}{2}}$	64,163	− 0·6	$^4P_{\frac{1}{2}}$	35,905	+ 0·3
$(s^2p^4.6p)$ $^4S^0_{1\frac{1}{2}}$	59,277	0			

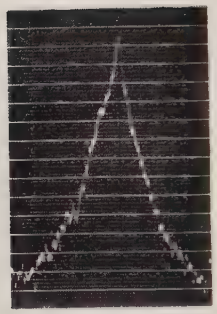
Table 3. Pressure displacements of the unclassified lines of the Xe II spectrum

Wave-length	Displacement (cm.$^{-1}$)	Wave-length	Displacement (cm.$^{-1}$)	Wave-length	Displacement (cm.$^{-1}$)
6994	− 0·8	4545·34	− 5·9	4238·37	− 8·2
6808	− 0·6	4541·03	− 4·5	4223·14	− 10
6705	− 1·0	4537·51	− 1·6	4209·75	− 6·0
6621	− 0·4	4537·02		4209·53	
6513	− 0·5	4486·12	− 0·5	4208·61	
6270	+ 0·6	4481·07	− 6·0	4193·25	− 7·0
6009	− 1·1	4462·38	− 7·7	4180·20	− 7·6
5616·99	− 0·3	4448·28	− 6·5	4158·14	− 5·1
5531·33	− 0·5	4395·91	− 6·0	4112·25	− 6·0
5188·28	large (−)	4393·34	− 5·4	4109·20	+ 0·6
5092·22	− 4·4	4369·34	− 5·4	4157·55	− 9·6
4853·90	+ 0·7	4337·14	− 0·9	3908·00	− 10
4773·34	− 0·7	4335·95	− 1·2	3849·97	− 8·2
4715·31	− 2·5	4330·63	− 7·6	3762·43	− 1·2
4698·20	− 3·0	4321·95	− 0·9	3720·93	+ 0·8
4668·72	− 1·2	4310·69	− 5·3	3717	− 1·5
4592·22	− 7·0	4296·52	− 2·9	3672·68	− 1·2
4585·65	− 6·2	4251·68	− 6·1	3612·52	− 0·8
4577·36	− 6·2	4245·54	− 7·3	3565·35	+ 0·8

a

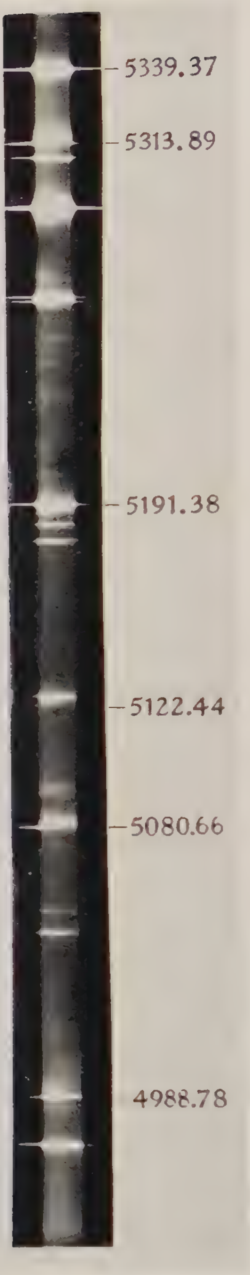


b



c

d



The majority of the Xe II lines observed in the present investigation are not included in the classification by Humphreys, de Bruin and Meggers. But all these lines are included in a list of Xe II wave-lengths published by Bloch, Bloch and Déjardin*. Table 3 lists the lines which have measurable pressure displacements. The wave-lengths are those given by Bloch, Bloch and Déjardin and are the wave-lengths in air.

Table 3 shows that, in a general way, the displacements increase as we pass toward the shorter wave-lengths. This is an indication that the displacements are greater for levels with outer electron orbits.

§ 5. DISCUSSION OF RESULTS IN THE Xe II SPECTRUM

It has been generally supposed that the pressure displacements observed in ionized-gas spectra arise from internal electric fields. The present results for the Xe II spectrum support this view, inasmuch as the term displacements are found to be larger for terms of higher principal quantum numbers, a rule which in general holds for Stark displacements. From the present results, moreover, it is possible to say that, if the displacements arise from electric fields, these fields must be very large. For, certain of the Xe II lines have been observed† in electric fields as large as 100,000 V./cm. and they exhibit no measurable Stark displacements. Among these, for example, was the line λ 6093·53 which exhibits, in the present investigation, a large displacement (5 or 6 cm.$^{-1}$) toward the red.

The discrepant displacements occurring among members of certain multiplets of Xe II form an exception to the rule found by Pretty to hold in the A II and Ne II spectra: but they do not disfavour the hypothesis that the displacements are essentially a Stark effect, for Stark displacements are known to be governed by the arrangement of the terms in the energy diagram.

§ 6. DESCRIPTION OF THE PLATE

(*a*) Shows the Xe I line, λ 6318·08 ($2p_8$–$6d'_4$), broadened in the high-pressure source in comparison with the unbroadened line in the low-pressure source; (*b*) is a microphotometer record of the broadened line showing the asymmetry of the broadening; while (*c*) shows the Stark effect for the same line in fields attaining a maximum of 60,000 V./cm. (*d*) Shows some typical displacements in the Xe II spectrum, the central strip representing the high-pressure source.

§ 7. ACKNOWLEDGMENT

The writer has great pleasure in acknowledging the kindly interest which Prof. A. Fowler has taken in this work.

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* *Ann. d. Phys.* (10), **2**, 461 (1924).

† H. W. Harkness and J. F. Heard, *loc. cit.*

DISCUSSION

Prof. A. O. RANKINE. I should like to raise the question of the use of the term "displacement" for the quantities recorded in the last columns of the tables, though I know that the author is not responsible for the choice of this nomenclature. The quantity has the dimensions of the reciprocal of a length, and the units are cm^{-1}. Scarcely anything could be more misleading than to call this a displacement when the same name in dynamics denotes a length, frequently measured in cm. The uninitiated would naturally take a pressure displacement to mean the shift of the spectral line on the spectrogram due to the application of pressure.

AUTHOR'S reply. If the term "displacement" must always refer to a length then there is no escaping the contradiction here which Prof. Rankine points out between the name and the unit. However, I cannot agree that the use of this nomenclature must be confusing to anyone with a knowledge of spectra. One usually thinks of a spectral line on an energy level as a particular position in a continuum whose unit is the wave-number or cm^{-1}, and quite naturally, I believe, one thinks of a change of position as a "displacement."

DEMONSTRATION

677.31

"Infra-red photography of black or pigmented animal fibres." *Demonstration given on June* 16, 1933, *by* P. W. CUNLIFFE, Ph.D., F.R.P.S., Wool Industries Research Association.

The microscopic investigation of black wool and other pigmented fibres presents considerable difficulty, and any method which assists in rendering the fibre detail more obvious is therefore welcome. An advance in this direction has been made possible, by the use of infra-red photography, in the laboratories of the Wool Industries Research Association.

The presence of black hairs in the fleece, the remnant of the old hairy outer coat of the primitive sheep, is a disadvantage in the manufacture of wool fabrics, and its hairy or kempy nature has hitherto been difficult to establish or examine.

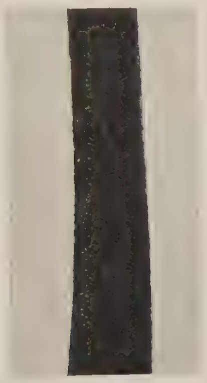

Figure 1. (× 165.)

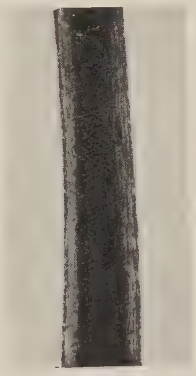

Figure 2. (× 165.)

The two photomicrographs reproduced herewith are of a black lamb fibre. Figure 1 is the rendering given by a panchromatic plate (Ilford S.G.) with a deep red filter (Wratten F.) and shows very little detail, giving an impression of an almost solid black filament, whilst figure 2 is from an infra-red photograph (Ilford plate and filter) and shows considerable detail. The fibre is medullated, and this indicates a kempy or hairy nature rather than a desirable wool characteristic. The photograph necessarily shows a black centre whatever the quality of the transmitted light.

Similar results have been obtained on other heavily pigmented fibres and hairs, such as kangaroo and negro hair.

PRESENTATION OF THE DUDDELL MEDAL, 1933

To Prof. WOLFGANG GAEDE

June 16, 1933

Prof. A. O. Rankine, President, in making the presentation said:

The Duddell Medal is awarded annually by the Council of the Physical Society to an inventor of physical instruments of outstanding importance and utility, or to a discoverer of new materials essential in their construction. There are no restrictions of nationality, and it is the duty of the Council each year to search the world to find the most worthy recipient.

The institution of the medal is comparatively recent, but I think it will be agreed that the short list of awards during the last decade has set a very high standard and that the illustrious names which it contains have been chosen well. To this list the Council with complete unanimity last December added another name, that of Dr Gaede whom we now welcome very warmly to receive the medal.

I am sure that it is unnecessary for me to explain the reasons for the choice which the Council has made. Gaede and vacuum are almost synonymous terms, and all physicists are well aware of his outstanding contributions to the art of vacuum-making and of how his wonderful pumps have facilitated physical investigations in innumerable directions.

In conclusion I must permit myself to remark that it has been a surprise and pleasure to discover that Dr Gaede is as young as he is. The name of his pumps has been so often on our tongues that most of us had perhaps begun carelessly to think of him as belonging to a previous generation. Now that we meet him in the flesh we rejoice to find ourselves mistaken, and we congratulate him on his comparative youth. We are sure that in the many years that lie before him his unsurpassed skill and ingenuity will continue to be at the service of physical science. With him in the forefront progress is certain to be made.

Professor Gaede, in the name of the Physical Society I ask you to accept this, the Duddell medal of 1933. It is a token of the esteem in which we hold you and your work. We desire to honour you as highly as we can, but we realize that in accepting our award you will be conferring on our Society a still greater honour.

Prof. Gaede, in accepting the medal, said:

It is hard for me to express in English how much touched and pleased I am to be in your midst and to be able to thank you in person for the great honour you have accorded me. This feeling is all the greater as in your award I see a practical sign of the international union of scientific work and of our co-operation. Therefore I express my heartiest thanks.

621.527.8

NOTE ON GAEDE PUMPS

In 1905 Gaede demonstrated at the scientific congress at Meran his newly produced air-pump, which in the main consisted of a porcelain drum divided into cells and rotating in an iron casing half filled with mercury. The advantages of this "rotary mercury pump" were simplicity of form, security, high vacuum and ten-fold pumping speed. It therefore gained widespread popularity in laboratories and in electric-lamp factories.

Then Gaede applied himself to constructing various rotary oil air-pumps which first were to serve as fore pumps for the mercury pumps, and then were used as independent pumps. The principle of all these pumps had been brought into use by Otto von Guericke. A chamber alternately increases and decreases in volume. When increasing it draws air from the receiver which is to be evacuated, and when decreasing it compresses the enclosed air and puffs it out into the atmosphere or into the fore vacuum. Till then all pumps for laboratories and engine works, except jet pumps, were based on this principle; it was Gaede who in the years 1910–1915 worked out two new principles for the construction of air-pumps: the friction principle used in the molecular pump and the diffusion principle.

On the friction principle the pumping effect arises by air friction on the smooth surface of a spindle rapidly rotating in the middle of a cylindrical casing. The circular space between the cylindrical spindle and the cylindrical casing is interrupted by a projection fixed on the inner wall of the casing, so that the spindle may rotate freely at a distance of about 0·1 mm. from the projection. The rotating spindle produces by gas friction an effect of pressure in the surrounding gas so that between the sides of the projection a difference in pressure of a few millimetres arises. The essential point is that this difference is independent of the pressure itself, according to the kinetic theory of gases. If we lower the pressure itself beneath the value of the difference, we gain a high vacuum and a great pumping speed of more than one litre a second. A special advantage of this molecular pump is that it produces a vacuum free from any vapour. Although it constituted a great scientific and technical advance it was prevented from playing its part in full by the inventor himself, who superseded it by a new construction based on the diffusion principle.

On the diffusion principle air-free mercury vapour, produced in a small vessel with a reflux condenser, streams rapidly through a pipe. Into this vapour-pipe leads a suction pipe which is connected with the vessel to be evacuated. Mercury vapour enters the mouth of the suction pipe where it must at once be condensed in order to keep the vessel which is to be evacuated free from vapour. At the same time air leaves the mouth of the suction pipe and enters the vapour pipe where it is seized by the streaming mercury vapour and carried to the fore vacuum. In the mouth of the suction pipe, as we see, air and vapour move in opposite directions. The essential point of the diffusion principle is that the air shall diffuse through the counter-moving vapour and not be pushed back by the vapour and thus prevented from entering the vapour pipe, as atmospheric air is pushed back by steam issuing from a boiler and is prevented from entering it. The difficult problem therefore was to bring about the diffusion of the air into the vapour and to avoid the prevention of its exit from the suction pipe. It was Gaede who solved this problem by rightly dimensioning the width of the mouth of the suction pipe. His experiments and calculations proved that the width of this mouth or slit must not exceed the free mean path of the gas molecules in the vapour at this point. This fundamental condition is fulfilled in all the high-vacuum vapour pumps which have subsequently been produced by different inventors and under different names. Not only the vacuum but also the pumping speed of these pumps is excellent. The reason lies in the correspondence between decrease of pressure and increase of the velocity with which the air diffuses through the vapour. In all these pumps, independently of their pumping speed, the velocity of the diffusing air is about 10 metres per second. The pumping speed is the product of this velocity and the area of the opening mentioned above. Gaede attained in his first pump a speed of about 0·1 litre per second and finished with about 400 litres per second. In his latest constructions he has used an improvement due to Langmuir, who unified Gaede's diffusion principle with the principle of the steam jet pump well known for its great speed. This improvement has been very generally introduced. Burch showed that the pumps might be worked advantageously with oil vapour of low pressure instead of mercury vapour.

REVIEWS OF BOOKS

Great Men of Science, by PHILIPP LENARD. Pp. xix + 389. (London: G. Bell and Sons, Ltd.) 12*s*. 6*d*. net. *Makers of Astronomy*, by HECTOR MACPHERSON. Pp. 244. (London: Humphrey Milford, Oxford University Press.) 7*s*. 6*d*. net.

Thumbnail biography is perhaps the most difficult of all the arts of the biographer, only to be approached at the other end of the scale by the intricacies of the task of transferring to a full-size canvas such an immense mass of heterogeneous material as confronted, say, Boswell or Lockhart. In each instance the individual traits, all the more interesting if they happen to be eccentricities, which elucidate character are to be sketched; in each instance the figure sketched must be placed against the background of his times, and his contribution to the advancement of the knowledge of his age duly explained and assessed. No easy matter, this, to achieve within the compass of a thousand pages—an almost impossible business to compress it within the limits of a thousand words. Nevertheless, many writers have attacked and accomplished such a task. What success has attended the effort of Prof. Lenard? He has, indeed, covered a most remarkable range—his roll of names opens with Pythagoras of Samos, closes with Friedrich Hasenöhrl and includes studies of sixty-five scientific worthies. Representatives of physical science naturally predominate, but he has not been unmindful of the claims of other branches of knowledge, and has provided us with sympathetic pictures of Darwin, Linnaeus, Dalton and Davy.

Perhaps the outstanding feature of his sketches is his quiet insistence on the difficulties which face the discoverer of a new concept—a matter all the more troublesome to make clear to the student of to-day to whom a concept novel and complex in 1831 is the merest commonplace a century later. Another valuable feature is provided in the intimate and informing sketches of such workers as Ohm, Oersted, Weber, Klaproth, Gauss and Carnot who, as far as their personalities are concerned, are names and nothing more to many English workers.

In brief notices, where every word must be weighed, an almost meticulous accuracy is demanded of the writer. It is unnecessary to stress the great care with which Prof. Lenard has weighed his words and the high standard of accuracy which he has set for succeeding writers. Nevertheless, the good Homer nods, on occasion, and Prof. Lenard leaves us with the impression that Copernicus swept away the whole complicated system of cycles and epicycles necessary to make a geocentric theory fit the facts of observation. Which he didn't; Copernicus was a thoroughly orthodox epicyclist, and the distinction of getting rid of that complicated theory is Kepler's. Nor was the charge of heterodox astronomical beliefs more than one charge among many that sent Bruno to the stake. The matter is not easy to elucidate, but Gaspar Schopp was present at the trial and his famous letter to Conrad Rittershausen gives as the damnatory clauses of the sentence Bruno's denial of transubstantiation and of the virgin conception, his publication of the *Bestia Trionfanti*, and lastly, his teachings concerning the lawfulness of magic, that Christ was not God, that the number of worlds was infinite and so forth. Bruno's astronomical teachings were among the least of the charges preferred against him. And did Joule die at Salford? But we must not seem ungrateful. Prof. Lenard has produced a series of charming and scholarly sketches which all may read with pleasure and with profit.

Dr Macpherson's task is easier; he has restricted himself to one field, that of astronomy, and he is consequently able to place his biographical studies in a more intimate relation to the majestic sweep of the development of his chosen science. He divides his studies into a series of well-marked sections—the pathfinders, Newton and after Newton, Herschel and his successors, pioneers of astrophysics, watchers of the skies, and explorers of the universe. His studies of the makers of astronomy open with the names of Copernicus and Tycho, close with those of Eddington, Shapley, van Rhijn and de Sitter. We may not accept his opening statement that "the first of the long list of great men who are entitled to the designation of makers of astronomy was born at Thorn on the Vistula on 19 February 1473," but our thanks are due to him for a learned and well-balanced sketch of the development of modern astronomy.

Both books are admirably produced and illustrated.

A. F.

Opticks, or a treatise of the Reflections, Refractions, Inflections and Colours of Light, by SIR ISAAC NEWTON. Pp. xxviii – 414. (London: G. Bell and Sons, Ltd.) 6*s*.

"Yes, my boy, the 1704 edition, not the suspicion of a crack in the joints, label intact, not a stain or a tear anywhere, in mint condition. And I picked it up for thirty shillings at Nith's." So might your dearest friend extol the beauties of his copy of the *Opticks* what time you remark, even as Johnson when peering at the beauties of Bennet Langton's grounds, "Non equidem invideo; miror magis."

The enterprise of the publishers has placed within the reach of those of us who have not been lucky enough to pick up a copy of the *Opticks* for thirty shillings, a means of making a first-hand acquaintance with the text of a very remarkable book. Nay, even the fortunate possessor of the 1704 edition may do well to purchase this handy volume. It will save him from committing the sacrilege of reading the original text.

The present edition is a reprint of the fourth edition (1730) "corrected by the author's own hand, and left before his death with his bookseller." The type is clear and pleasant to read, the illustrations well reproduced, the format a small octavo; and we have a foreword by Einstein and an introduction by Whittaker. Need one say more?

A. F.

The Anatomy of Modern Science, by BERNHARD BAVINK. Translated from German by H. STAFFORD HATFIELD. Pp. xiii – 683. (London: G. Bell and Sons, Ltd.) 21*s*.

More than a generation ago there issued from the press a work of great insight and magistral scholarship, *A History of European Thought in the Nineteenth Century*. That book influenced many workers powerfully, not so much by any profundity of original contribution as by the clarification of ideas resulting from the study of a masterly survey of all branches of thought. The mantle of John Theodore Merz has fallen upon Dr Bavink, who has produced a volume destined, we trust, to aid many who regard with growing bewilderment the ever-accelerating rate of advance of the sciences, physical and biological.

Dr Bavink's volume is divided into four sections. The first part deals in considerable detail with the facts and philosophy of modern physical science, the second part with "cosmos and earth." In the third part the facts and problems of biological science are handled and, just as part 2 places the notions of modern physical science in their relation to the cosmos, so part 4, on "nature and man," considers the cosmical implications of the facts of biology.

The book is not one to be read uncritically. We cannot, for example, subscribe to Dr Bavink's criticism of the conceptualist position as expressed in his italicized statement that "*Atoms are just as real things as cannon balls or grains of sand, as waves on water or mountains*." They remain what they always have been, integral parts of a conceptual world framed to represent the facts of sense-presentation. And if this concept is valid as an explanation of the pressure read on a Schräder gauge, it remains valid as an interpretation of that mark on a photographic plate known as an α-ray track. Multiply such phenomena by the million, and the atom remains just as much or as little "real" as it was in the days of Mach.

As we have remarked, the book must be studied with one's critical judgment ever alert. So read, it is a most stimulating and valuable contribution to modern thought.

The book is excellently produced, and its seven hundred pages do not lie over heavily in the hand.

A. F.

Matière et Energie, by V. HENRI. Pp. 436. (Hermann, Paris, 1933.) 110 fr.

Prof. Victor Henri has given here a good and not difficult account of the present state of atomic physics, bringing the information up to the most recent discoveries about the nucleus. The book will be found specially convenient for the experimental physicist, who does not so much want detailed information about the abstruse mathematics of the new mechanics as a general review of the present state of knowledge. There are a great many useful tables of atomic constants of all kinds and there is a good deal of mathematics, but mostly of a simple type. The work is treated in a historical manner. The first chapter describes the evidence for the existence of atoms—Brownian movement and so on—and describes the various ways in which they can be counted. The next three chapters deal with the chemical elements, their X-ray spectra, the periodic system, atomic numbers, etc., and the next three with radioactivity, isotopes, nuclear disintegration and neutrons. There is then an account of the special properties of atoms, as revealed by X-ray crystal analysis and by gaseous viscosity. In the last chapter the structure of atoms is dealt with, mostly by the use of the old mechanics, though the new is sufficiently developed to explain the changes. Altogether the book should prove most useful for any one who wants to learn the main points of interest in modern physics, and the large number of tables given will also make it useful as a work of reference.

The Classical Theory of Electricity and Magnetism, by MAX ABRAHAM. Revised by Prof. RICHARD BECKER. Translated by JOHN DOUGALL, M.A., D.Sc., F.R.S.E. Pp. xiv + 285. (London: Blackie and Son, Ltd.) 15*s*.

"Abraham-Föppl" needs no introduction to European physicists—its merits have been known since 1894 to all interested in the development of Maxwell's theory. In the eighth German edition, of which the present volume is a translation, a number of alterations and additions have been made which, however, have left essentially unchanged the main features of a work which has become a classic. At the present juncture it is interesting to note that Prof. Becker remarks "In the choice of units...the system used throughout is the Gaussian system in which the energy density in a vacuum is equal to

$$\frac{1}{8\pi}(E^2+H^2)\ \text{ergs/cm}^3.$$

And the dielectric constant and permeability of a vacuum are each taken as unity. It does not seem possible at present to set up a system of units which will satisfy the electrical engineer and the physicist alike." The volume is divided into four parts. Part 1 is an admirable treatment of vectors and vector fields, part 2 deals with the electric field, part 3 with the electromagnetic field and part 4 with energy and forces in Maxwell's theory.

The translation flows easily, and it goes without saying that the book is admirably produced.

A. F.

Relativité Générale; Gravitation et Électricité; Cosmologie, by A. EINSTEIN. Pp. 109. (Hermann, Paris, 1933.) 35 fr.

This is a French translation from the German of three essays by Einstein. The first, which fills nearly two-thirds of the book, is devoted to the relativistic theory of gravitation. It follows the ordinary routine for the subject, developing the mathematical theory of tensors; it makes specially clear the distinction between the parts of the theory concerned with free space and the interior of matter. The second essay develops Einstein's new unitary theory, depending on the idea of "parallel displacement," and succeeds in doing it in only 26 pages. The whole theory is there, and the account would be useful to any one already fairly familiar with the ideas of advanced geometry, but it is to be doubted whether the beginner would make much of such a condensed account of a very abstract theory. The last essay is much more physical in its thought, and devotes 10 pages to a very good account of the theory of the expanding universe. The mathematical formulae contain rather too many misprints.

Théorèmes de Conservation dans la Théorie des Chocs Électroniques (Exposés de Physique Théorique IX), by A. GOLDSTEIN. Pp. 26. (Hermann, Paris, 1933.) 7 fr.

In the developments of wave mechanics there have been a good many of the simpler points which have been incompletely expounded, though it was always evident that they were completely in order. To any one familiar with the new mechanics the present work will hardly be necessary, but for a beginner it may be a convenience to see how the conservation theorems for collisions formally emerge from the theory.

Actualités Scientifiques et Industrielles. (1) No. 56, *Sur la Théorie du Rayonnement, d'après M. C. G. Darwin*, by E. NÉCULCÉA; pp. 24; 7 fr. (2) No. 57, *Sur l'Absorption Exponentielle des Rayons β du Radium E*, by G. FOURNIER and M. GUILLOT; pp. 38; 10 fr. (3) No. 59, *La Diffraction de la Lumière par des Ultra-Sons*, by L. BRILLOUIN; pp. 32; 10 fr. (Paris: Hermann et C^{ie}, 1933.)

(1) The first of these three numbers of the well-known French series of short monographs is a translation, rather than what we understand by an "analyse détaillée," of a recent paper by C. G. Darwin*. It will doubtless be of service to French readers, but most English readers will not unnaturally turn to the English original.

(2) In the second, MM. Fournier and Guillot present a critical discussion of earlier work on the absorption of β-rays by matter, together with an account of their own more recent work carried out at the Laboratoire Curie in Paris.

(3) In this paper M. Brillouin discusses in some detail the theory of the very interesting diffraction phenomena which are observable when light traverses a medium in which high-frequency elastic waves (supersonic waves) are maintained.

H. R. R.

Optik: Ein Lehrbuch der elektromagnetischen Lichttheorie, by Prof. MAX BORN. Pp. vii + 591; 252 figures. (Berlin: Julius Springer, 1933.) RM. 36; gebunden RM. 38.

In his presidential address to the Optical Society three years ago Mr Twyman pointed out that optics should no longer be defined as "a science which treats of light and the phenomena of vision," but rather as the science which deals with radiation, and he proceeded to show how some of the more recent developments of physics, such as wireless, X-rays, positive rays, etc. could with justice be included in the "optical empire." Although we have not reached the stage when a book is entitled, say, "Optics: Wireless Valve Transmitters," it is desirable that authors of books on optics should specify the branches with which they are dealing. Prof. Born has done this in the book under review, and in his preface he is careful to state quite specifically the limits within which he has confined the treatment of his subject and his reasons for choosing these limits; the choice appears to be a wise one. After a brief historical survey occupying eight pages, he divides the subject into eight sections: the electromagnetic theory of light for transparent isotropic substances without dispersion; geometrical optics; interference; diffraction; crystal optics; metal optics; molecular optics; emission, absorption and dispersion. The method of treatment is almost entirely mathematical, but can be followed by any one possessing a good knowledge of the theoretical aspects of optics. In his preface Prof. Born remarks that Drude's well-known book is one of the few scientific books which will always remain a classic. If this is true, the present book is also certain to rank as a classic. The author is to be congratulated on the clarity of his style and the publishers on the excellent manner in which they have produced the book.

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* *Proc. R. S.* A **136**, 36 (1932).

Actualités Scientifiques et Industrielles, No. 54. *Spectres d'Absorption Visibles et Ultra-violets des Solutions: Technique de leur emploi au laboratoire de Chimie*, par M. CHATELET. Pp. 24. (Paris: Hermann et C^{ie}, 1933.) 7 fr.

This monograph, the latest in a noteworthy series, presents very clearly and concisely the essential points in the photometry of absorption spectra, with particular reference to the methods adopted by the author himself. In addition to brief descriptions of certain apparatus, e.g. a water-cooled hydrogen tube as a source of ultra-violet continuum, several devices for diminishing spectrographic intensities in known ratios, and a microphotometer, the text includes a few useful notes on other practical matters, such as cements for absorption cells, choice and development of photographic plates, adjustment of apparatus, and the like. There is little or nothing to criticize, but it may be pointed out that twice on p. 3 the word "term" (*terme*) is wrongly used for "line" (*raie*). W. J.

The Practice of Spectrum Analysis with Hilger Instruments, sixth edition, compiled by F. TWYMAN, F.R.S. Pp. 58. (London: Adam Hilger, Ltd., 1933.) 3*s*. 6*d*. net.

The fifth edition of this useful and attractive booklet* is followed by a sixth after an interval of only twenty-one months. The size of the manual and the titles of the chapters are little changed, but the text has been very thoroughly revised by Dr A. Harvey, whose name and Mr E. S. Dreblow's are now added to the list of names of contributors. Space has been found for much useful new matter by rearrangement and condensation of older material and the deletion of a few sections whose period of usefulness is now over. For example, the bibliography which formed the third and last appendix in the fifth edition is now omitted, and references formerly included in it and in the text are now given in footnotes. Much of the change occurs in the chapter on quantitative spectrum analysis, which now contains discussions of the application of logarithmic wedge sector, the ratio powder method, etc. In other chapters new matter occurs in the form of notes on preliminary treatment of specimens, the effects of polarity in arcs between poles of unlike composition, the characteristics of photographic plates, etc. W. J.

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* Reviewed in *Proc. Phys. Soc.* **44**, 108 (1932).

Spectroscopy in Science and Industry, by S. JUDD LEWIS, D.Sc., F.I.C., Ph.C. Pp. viii + 94 and 10 plates. (London and Glasgow: Blackie and Son, Ltd., 1933.) 3*s*. 6*d*. net.

This is an "introductory manual" in Blackie's *Technique Series* describing the applications of spectrographic methods in industrial and other practical problems, especially the qualitative and quantitative spectrum analysis of substances. It is not concerned with the description and analysis of spectra, the subject for which Sir Arthur Schuster proposed the name "spectroscopy." The instruments described are all by Messrs Adam Hilger, Ltd. Some of these are shown in figures in the text and in plates; in the rest of the plates spectrograms taken with Hilger instruments are beautifully reproduced. In many respects the text is disappointing, although it has some interesting sections. If the length of this review permitted, it would be only too easy to cite many instances of unordered treatment, loose expression and actual mis-statement, which the elementary and empirical nature of the descriptions can hardly be said to excuse. Compared with such blemishes, the appearance of "ultimate rays" on pp. 29 and 35 as a rendering of "*raies ultimes*," and "Molesey" on p. 88 for "Moseley" are mere trifles.

W. J.

High-Frequency Measurements, by A. HUND. Pp. xi + 491, with 373 diagrams. (McGraw-Hill Book Co., London and New York, 1933.) 5 $.

The work is divided into eighteen chapters, of which the first three serve as a general introduction to the subject, giving summaries of fundamental relations, of the more important properties of high-frequency circuits, and of methods for the production, detection and measurement of rapidly alternating currents, with other information of a general nature. Of the remaining fifteen chapters, each one as a rule deals with the measurement of one particular quantity—voltage, frequency, capacitance and so forth—or of sets of closely associated circuit constants.

A good selection of methods is given, and a compilation of this kind, in a subject of which the literature is so widely scattered in scientific and technical journals, cannot fail to fulfil a very useful purpose. It should be valuable not only to all engaged regularly in high-frequency measurements, but also to the large general body of physicists with no more than an occasional interest in such work. Readers of the latter class in particular will find the subject-matter admirably arranged for easy reference.

The alphabetical index (27 pp.) is if anything too grimly comprehensive; thus, a brief section of a minor footnote (on p. 35) is indexed no fewer than six times, under "Bohr, energy levels," "Born energy levels," "Heisenberg, W., on spectroscopy," "matrix calculus," "Pauli, on spectroscopy" and "spectroscopy"—roughly one reference to each line of the text—but this is rather an extreme example. A few slips and misprints point to a rather hurried completion of the text—the references to one "K. Omnes," a worker on electrical conductivity of metals at low temperatures, provide one of the more striking examples of this.

In the second edition, which will doubtless be called for very soon, it would be a boon to readers of subnormal, and even of normal, eyesight if the otherwise excellent diagrams could be printed on a larger scale, even though this could not be done without an appreciable increase in the size and price of the volume.

H. R. R.

Gas Analysis by Measurement of Thermal Conductivity, by H. A. DAYNES, D.Sc. (Cambridge University Press.) 16*s*.

To certain present-day scientific workers the term "katharometer" bears something of the significance to the old lady of that blessed word Mesopotamia—high sounding and comfortable but deeply mystic in meaning. It conjures up to their imagination the key to the solution of the most difficult measurements.

A glance through the present volume certainly impresses one with the versatility of the "katharometer" as a tool in the hands of the physicist, chemist and biologist. We are told that by means of it the quantity of carbon dioxide given off in the respiration process of a fly can be measured, and that this amounts to about half a cubic centimetre in six hours. It is, however, in the industrial field of gas analysis that the "katharometer" has come into its own, and according to the author of this volume many thousands of these instruments are in use for the measurement of the composition of flue gases from power house boilers.

Yet the hot wire method is not without its drawbacks, and Dr Daynes has done a real service by writing this comprehensive treatise on the instrument. His material has largely been drawn from experience in applying this method of gas analysis to practical problems and the fact that the manufacturers of the instrument in this country have allowed him unrestricted use of information obtained in the course of development work reflects great credit on them for their public spirit.

In his historical introduction the author traces the early stages of the development. It appears that Leon Somzee originated the device of analysing a gas by determining its conductivity about 53 years ago, but it did not reach the stage of successful application for many years. Other names that stand out conspicuously for their contributions towards making the method a practical success are those of Koepsel and Shakespear.

A student contemplating using this method in an investigation will find the volume a mine of information, for it deals with all aspects of the subject, including accessory apparatus. A feature of interest in the book is the description given of efforts to circumvent the limitation imposed by the fact that gases do not differ very greatly in thermal conductivity except in the case of hydrogen and helium.

The printing of the book and the illustrations are admirable and reflect credit on author and publishers.

EZER GRIFFITHS

Collision Processes in Gases, by F. L. ARNOT. Pp. viii + 104. (London: Methuen and Co., 1933.) (Methuen's Monographs on Physical Subjects.) 3*s*.

The book deals mainly with work in which individual collision processes are more or less directly studied. Part 1 discusses collisions between electrons and atoms, part 2, in two short chapters, collisions between photons and atoms, and between normal, excited, and ionized atoms. This deliberate limitation of the thesis has made it possible to give, in one small volume, a connected account of a fairly wide range of the more important and more readily interpretable of collision phenomena.

Students reading for honours degrees in physics will find here concise accounts of such topics as critical potentials, probabilities of excitation and ionization by electrons, the Ramsauer effect, and the numerous applications of Klein and Rosseland's "collisions of the second kind"—all well adapted to serve as convenient introductions to more detailed study. As would be expected, Dr Arnot, within his obvious limitations of space, deals particularly fully with the recent beautiful work on the scattering of electron beams by gases at low pressures.

The book is to be recommended as a valuable and timely addition to this useful series of monographs.

H. R. R.

Experimental Atomic Physics, by G. P. HANWELL and J. J. LIVINGOOD. Pp. xiii + 472. (London: McGraw-Hill Publishing Co., Ltd.) 30*s*.

This work is a very important contribution to pedagogical literature. The authors have concentrated on the experimental side of their subject, and their book contains descriptions of an admirable collection of laboratory exercises in atomic physics which should prove of great value alike to teacher and to student.

The book is, however, much more than a laboratory manual. It is an introduction to the subject, presented from the experimental side and intended to be read by those who have already had a course of elementary training in the classical aspects of physics. Although "a working knowledge of the calculus and of differential equations is... presupposed" the mathematics employed in the book is quite elementary in character. The topics treated of are developed logically and carefully and are, in the order of presentation, radiation, the atomicity of matter and electricity, the wave aspect of matter, thermionic and photoelectric effects, spectra, X-rays and radioactivity.

The book may be recommended unreservedly.

An Elementary Introduction to Physics, by EDGAR BOOTH. Pp. xvi – 465. (Glebe, N.S.W.: Australasian Medical Publishing Co., Ltd. London: H. K. Lewis. 1932.)

The author in his introduction indicates a feeling that his book will be criticized as not being up to date. It is certainly in the older tradition, many of the methods and verbal illustrations being reminiscent of a former generation of text-books. The study of mensuration and attention to accuracy of measurement are emphasized; mass and weight are not confused in one hybrid formula; proofs tend to be algebraic rather than geometrical; magnetism and electricity are the last subjects treated and together occupy one-ninth only of the book; modern physics is dismissed, if not with a shrug, at least with an electron and a paragraph; the cartesian diver reascends; levers are in systems one, two and three; the Grove cell still has its place between Daniell and Bunsen. Both the strength and weakness of the older tradition are present, but the virtues predominate. The general impression is that of sound teaching, and any student who knows the book well is well prepared to enter upon a course of London Intermediate standard. Upon general and historical topics the text is readable and informative. In this country school teachers may not discover anything very new in the book; but as an auxiliary text-book for those university students—and they are many—who read for science or medicine without previous acquaintance with physics the book may very well be recommended. J. P. A.

Examples in Physics, by W. G. DAVIES. Pp. 128. (London: Edward Arnold and Co.)

This little book marks, in one way, a further stage in the evolution of its species. With regard to subject divisions the old books were casual, while their descendants developed definite sections devoted to the main branches of physics. The book shows, in addition, clear and adequate subdivisions. As a result, provided unconventionality is not sought, an example of any desired type may be found very speedily in one of the 25 subsections on general physics, the 12 subsections on heat, or elsewhere. The 580 examples themselves, many of which come from examination papers (although the source of individual questions is not indicated) are well up to the standard of Higher Certificate and London Intermediate examinations. They do not consist solely of numerical examples. A noteworthy, and on the whole up to date, collection which may be recommended to teacher and student.

J. P. A.

(1) *Professional Papers of the Air Survey Committee*, No. 8. *A Simple Method of Surveying from Air Photographs*. By Lieut. J. S. A. SALT, R.E. Pp. 145; 46 figures, 5 plates and 4 sheets. 4*s*. net. (2) *Parallax Tables*. Supplement to the above. Pp. 82; 2 figures. 1*s*. 6*d*. net. (London: H.M. Stationery Office, 1933.)

(1) In a short preface Col. Macleod, the Chairman of the Air Survey Committee,

points out that the type of aerial survey to which the Committee has given most attention is that which aims at the production of "fully contoured maps of moderate though sufficient accuracy, based on a limited ground control and making use of comparatively simple instruments." The previous seven Professional Papers published by the Committee described from a somewhat theoretical viewpoint the technique developed: the present paper presents a more practical description. It is divided into thirteen chapters entitled "introduction," "characteristics of air photographs," "photography," "ground control," "stereoscopy," "effect of small tilts on vertical photographs," "minor control plotting," "compilation of blocks," "detail plotting," "contouring," "reproduction," "rapid map production" and "mosaics." There is also an index giving a summary of terms used in aerial survey, with references to the Professional Papers. The monograph is well written and is sure to prove of great value to those who are concerned with the practical aspects of aerial survey.

(2) The parallax tables have been compiled to simplify the computation of the relationship between parallax and heights when contours are being plotted with the aid of a simple stereoscope.

The Scientific Journal of the Royal College of Science, vol. III. Pp. 120. (London: Edward Allen and Co.) 7*s*. 6*d*.

With the appearance of this, its third annual volume, the *Scientific Journal of the Royal College of Science* may be regarded as well established. It gives varied and good fare to its readers, and we trust that it will attain the wide circulation which it merits.

The present volume, which is divided into three sections, offers to the chemists five papers dealing with surface reactions, chemical synthesis in plants, the treatment of mineral oils, the structure of dyestuffs and the contribution of the chemist to industry. The natural-history section has two papers, one dealing with hibernation and the other with the development and classification of vegetation. In the physical section are contributions concerned with loud-speaker diaphragms, the disintegration of atoms by fast protons, the uses of high speed cathode rays, and uniform columns in electric discharges. The names of the distinguished authors of many of the papers are sufficient guarantee of the critical and scholarly treatment of the various themes.

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